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Accelerated sample average approximation method for two-stage stochastic programming with binary first-stage variables

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A B S T R A C T

This paper proposes an accelerated solution method to solve two-stage stochastic programming problems with binary variables in the first stage and continuous variables in the second stage. To develop the solution method, an accelerated sample average approximation approach is combined with an accelerated Benders' decomposition algorithm. The accelerated sample average approximation approach improves the main structure of the original technique through the reduction in the number of mixed integer programming problems that need to be solved. Furthermore, the recently accelerated Benders' decomposition approach is utilized to expedite the solution time of the mixed integer programming problems. In order to examine the performance of the proposed solution method, the computational experiments are performed on developed stochastic supply chain network design problems. The computational results show that the accelerated solution method solves these problems efficiently. The synergy of the two accelerated approaches improves the computational procedure by an average factor of over 42%, and over 12% in comparison with the original and the recently modified methods, respectively. Moreover, the betterment of the computational process increases substantially with the size of the problem.

1. Introduction

Stochastic programming is a well-known modeling framework for optimization problems dealing with uncertainty. For the real-world optimization problems, which explicitly contain uncertain parameters, stochastic programs are much more versatile than the deterministic formulation. The two-stage stochastic program with recourse is mentioned as a general-purpose method to deal with uncertainty in the model parameters. In this important class of stochastic models, the first-stage decisions have to be made prior to knowing the realization of the uncertain parameters. The second-stage decisions are made once the problem data have been realized. The overall objective is to minimize the sum of first-stage costs and the expected value of the random second-stage or recourse costs. Such problems have been developed in extensive applications, e.g., production planning [1], capacity planning and resource acquisition [2,3], facility location [4], scheduling [5,6], vehicle routing [7,8], environmental control [9] and healthcare optimization [10].

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An ample applied stochastic model is the two-stage stochastic linear program with recourse. Occasionally, a general two-stage linear problem incorporates the integer condition to some variables in either the first stage or the second stage. In many practical situations, only the first-stage variables are restricted to be integer. The case of first-stage binary variables has been applied to solve a variety of stochastic problems such as plant location and routing, supply chain planning, capacity planning and capital investment. An integer L-shaped method developed for problems with binary first-stage variables and arbitrary second-stage variables by Laporte and Louveaux [11]. Their method extends earlier work by Wollmer [12] for two-stage stochastic linear programming with 0–1 integer first-stage variables. Thereafter, Laporte et al. [13] and Alonso-Ayuso et al. [14,15] applied a Branch-and-Cut and a Branch-and-Fix Coordination methodology for such problems, respectively.

The two-stage stochastic integer programming is known as one of the most challenging optimization problems since they integrate discrete optimization problems into the large dimensional stochastic programs. It is well known that many discrete optimization problems can be complicated to solve. Furthermore, a most difficult step in solving two-stage stochastic programs is the evaluation of the second-stage expected value caused by a large number of scenarios. A number of sampling based approaches have been proposed to estimate the second-stage expected value function.

The sample average approximation (SAA) method is an approach for solving stochastic optimization problems by using Monte Carlo simulation. In this method, a random sample is generated according to a probability distribution, and then the expected value function of the stochastic problem is approximated by the corresponding sample average function. The deterministic equivalent of the sample average approximation problem which specified by the generated sample is then solved by deterministic optimization technique. This procedure is repeated by generating several samples to obtain candidate solutions along with statistical estimates of their optimality gaps.

Over the last decades, the SAA technique has been used in various manners with different names. This method has been proposed to solve stochastic linear programs, among others, by Shapiro [16], Shapiro and Homemde-Mello [17], and Mak et al. [18]. The theoretical principles as well as the convergence properties of the SAA technique were studied in Shapiro and Homem-de-Mello [19], and Shapiro [20].

The SAA approach has been developed to stochastic discrete optimization problems in Kleywegt et al. [21]. In their study, the convergence rate and computational complexity of this procedure are reported to the two-stage stochastic programs with discrete first-stage decisions. Computational studies that take advantage of the SAA method to solve stochastic linear programs are described in Linderoth and Wright [22], and to solve stochastic routing problems in Verweij et al. [23]. Various investigations into the applications of SAA include logistics network design, supply chain design and planning, and healthcare optimization have been largely reported in the literature [24–31].

Because of the independence between the sampling and the optimization, the sampling technique and the optimization method can be separated into modules making the programming effort easier to manage. Even after the expectation has been approximated using the SAA technique, the computational burden of solving the resulting mixed integer linear programming (MILP) problems may still be prohibitive. For two-stage stochastic programs with recourse, there are several accessible methods to overcome this challenge based on the foundation of Benders' decomposition [32]. Most of these methods can be viewed as extensions of the L-shaped algorithm first proposed by Van Slyke and Wets [33].

In this paper, we study the two-stage stochastic programming problems with binary first-stage and continuous second-stage decision variables. The computational complexity of such problems still remains the major difficulty. The main contribution of this paper is to introduce an accelerated sample average approximation (ASAA) method to overcome the computational challenges inherent in solving these stochastic problems efficiently. In the developed solution method, an accelerated sampling-based approach is proposed to improve the main structure of the SAA technique. In the developed ASAA method, the proposed accelerated Benders' decomposition approach by Mohammadi Bidhandi et al. [34] is utilized to solve the MILP problems with binary variables. The efficiency of the accelerated Benders' decomposition approach has been examined in Mohammadi Bidhandi and Rosnah [28] to solve the integrated supply chain planning under uncertainty using the original SAA technique. The proposed ASAA method in this paper improves the main structure of the sampling strategy as well as the MILP solution phase.

As the computational experiments, we study the proposed modeling framework for the logistics network design problems by Cordeau et al. [35], which is developed to the stochastic supply chain network design (SSCND) problems in Mohammadi Bidhandi and Rosnah [28]. The computational study discussed later in the paper, shows that the synergy of the two improved approaches expedites the computational process significantly thus allowing larger, more realistically sized problems to be solved. Moreover, the optimality gap between the optimal solution and that derived by the ASAA and consequently the difference between the objective values has been significantly reduced.

2. Problem statement

The SAA technique has a modular structure as shown in Fig. 1. In the SAA computational procedure, firstly, M independent samples are generated, each of size N. Then, the SAA problem (as the MILP problem) has to be solved M times to find a lower bound on the objective value of the original problem. This step is the most computationally difficult step, as solving the M stochastic programs with N scenarios in each replication is very time consuming. In order to improve the SAA procedure, two major challenges need to be addressed:

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