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A frequency criterion for doubly clamped beam-type N/MEMS subjected to the van der Waals attraction

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ABSTRACT

In general, the vibration frequency of electrically actuated nano/micro-beams under suddenly applied DC voltage is decreased with an increase of the input voltage. However, this descending behavior may be changed, if the initial gap between the movable and fixed electrodes is increased or a large axial compressive residual stress is applied to the system. Accurate determination of the threshold of this frequency behavior change under the van der Waals (vdW) attraction is the objective of the present paper. To this end, a geometric non-linear Euler–Bernoulli electro-mechanical beam model which accounts for the effect of axial residual stresses and the vdW attraction is considered. The oscillatory behavior of the system is studied semi-analytically through the homotopy analysis method (HAM). Employing the HAM solutions, the threshold of the frequency behavior change of the system is obtained. It is found that there exist linear relationships between the normalized parameters of the problem at this threshold. Using this important finding, a frequency criterion for doubly clamped beam-type N/MEMS is introduced. The present findings agree excellently with available experimental and other existing results in the literature as well as those obtained by three-dimensional finite element simulations carried out in COMSOL Multiphysics commercial software.

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1. Introduction

Nano/micro-electro-mechanical systems (N/MEMS) are mostly used as sensors and actuators. Because of their small size, low power consumption, reliability and their capability of batch fabrications, they have found lots of potential applications in engineering. Clamped–clamped nano/micro-beams represent major structural components and play crucial roles in these systems. One of the most important phenomena associated with electrostatically-actuated N/MEMS is pull-in instability which occurs when the input voltage exceeds its critical value. In this case, the movable part of the structure suddenly collapses toward the substrate. At the first time, Nathanson et al. [1] and Taylor [2] simultaneously observed pull-in instability in micro-systems. Since most of the N/MEMS devices work on the basis of this unstable behavior of electrically actuated nano/micro-beams [3,4], to date, many researchers have been motivated to develop some mechanical models to investigate it. Some of them utilized the equivalent mass-spring model [5–7] and the others employed the continuous beam theories [8–16] to represent a more accurate description for it.

Besides pull-in analysis, analyzing the oscillatory behavior of nano/micro-resonators which have applications in many N/MEMS devices such as timing ones is quiet useful. Tilmans and Legtenberg [17] studied the free vibrations of an

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electrically pre-deformed wide double clamped micro-beam using the linear beam theory. They also validated their findings with experimental results. Abdel-Rahman et al. [18] investigated the oscillatory behavior of micro-beams utilizing the geometric non-linear Euler–Bernoulli beam theory in which the effect of midplane stretching had been taken into account. They investigated the static pull-in instability and the frequency of vibrating micro-beams about their static deflection numerically using the shooting method. But their solution did not converge for micro-beams with large initial gaps. Batra et al. [19] removed the incapability of Abdel-Rahman's model by presenting a simple and computationally efficient single-term Galerkin's approximation for the problem. It is noteworthy that, besides the free vibration analysis of electrically pre-deformed micro-systems, analyzing the oscillatory behavior of nano/micro-structures subjected to suddenly applied DC voltage has lots of application in N/MEMS resonators. This type of devices is also investigated by some researchers to date. Moghimi Zand et al. [20] presented analytical solutions to non-linear vibration of micro-beams under suddenly applied DC voltage. They approximated the electrostatic force using Taylor's series expansion which was not accurate enough for near pull-in cases. Fu et al. [21] presented an analytical approximation to non-linear vibration of an electrically actuated micro-beam. They solved the resultant non-linear ordinary differential equations, which is obtained by the pre-multiplication of the reduced governing equation of motion by the denominator of the electrostatic forcing term, thorough the energy balance method. Qian et al. [22] presented a similar work using the homotopy analysis method (HAM). The latest approach could predict the non-linear frequency of an electrically actuated micro-beam accurately even for the case of large amplitude of vibrations.

At sub-micron scales, the intermolecular dispersion forces such as Casimir and vdW attractions besides the electrostatic actuation influence the motion of N/MEMS devices. The vdW force arises from the correlated oscillation of the instantaneously induced dipole moments of the atoms placed at the close parallel conductive plates [23]. The vdW force is a short range force in nature, but it can lead to long range effects more than $0.1 \mu\text{m}$ [24]. The Casimir force can be simply understood as the long range analog of the vdW force, resulting from the propagation of retarded electromagnetic waves [25]. The effect of the vdW and Casimir forces on the response of nano/micro-beams has been investigated by many researchers to date [26–36]. However, the number of works devoted to vibrational analysis of such systems is very limited. Jia et al. [37] studied free vibrations of electrically pre-deformed functionally graded nano/micro-beams subjected to the Casimir force. They employed the differential quadrature method (DQM) and investigated the combined effects of axial residual stresses, geometric non-linearity and solved the problem for beams with some different boundary conditions. It is noteworthy that they reached a valuable conclusion in their work about the frequency behavior in systems without the effect of axial force. According to their report, the non-linear frequency of vibration mutually decreased with an increase of the input voltage. However, if the initial gap of the system increased, this type of behavior will be changed. Jia et al. [37] reported the minimum initial gap in which the non-linear fundamental frequency of vibrations may increase by making an increase in input voltage. It is noted that, this worthy conclusion has been reported in some other previous works [18,19], but none of them have not presented any criteria for predicting the threshold of the frequency behavior change in beam-type N/MEMS.

The main object of the present paper is to provide a criterion for predicting the type of frequency behavior in doubly clamped beam-type N/MEMS under the vdW attraction. To this end, a geometric non-linear nano/micro-electromechanical beam under the combined effects of axial residual stresses and the vdW attraction is considered. Employing the Galerkin weighted residual method, the governing partial differential equation (PDE) of motion is reduced to a set of non-linear initial value problems. Afterward, since accounting only for the first mode of vibration in the Galerkin method may provide high accurate results [16,21,22,38], the resulting set of non-linear ordinary differential equations (ODEs) are simplified to a single one. This non-linear ODE is solved semi-analytically using the HAM. The HAM is also optimized to accelerate the convergence of the homotopy-series solution for the case of large amplitude of vibrations. The present findings are compared and successfully validated with available empirical and other existing results in the literature as well as those obtained through three-dimensional (3-D) finite element (FE) simulations performed in COMSOL Multiphysics commercial software.

Utilizing the present HAM solutions, it is found that there exists a linear relationship between the normalized parameters of the problem at the threshold of the frequency behavior change of the system. Based on this important finding, the present frequency criterion is introduced by fitting a plane to the HAM results in the 3-D space at this threshold. The accuracy of the present criterion is also verified through direct comparison between its predictions and those reported in the previous studies.

2. Problem formulation

A schematic of a doubly clamped electrically actuated nano/micro-beam subjected to the vdW attraction is presented in Fig. 1. The length, width, and density of nano/micro-beam are L , b , and ρ , respectively. The initial gap between the non-actuated beam and the stationary electrode is d . Also, x , y and z are the coordinates along the length, width and thickness, respectively. W , t , I , ν and E respectively refer to the beam's deflection, time, second moment of the cross sectional area, Poisson's ratio and the effective modulus of elasticity which is replaced by $E/(1-\nu^2)$ when $b > 5h$ based on the plane strain theory [10].

Based on the Hamilton principle, the geometric non-linear governing equation of motion takes the form [4,12,13]:

$$EIW'''' + \rho b h \ddot{W} = \left(F_r + \frac{Eb h}{2L} \int_0^1 W'^2 dx \right) W'' + F_{es} + F_{vdW}, \quad (1)$$

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