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City expansion model based on population diffusion and road growth

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ABSTRACT

In this paper, a city expansion model is proposed to capture the coevolution relationship between population diffusion and road growth. In the model, we adopt the physical diffusion process, which considers the influence of the road network topology and random exploration factor, to analyse the population diffusion based on the cellular automata (CA) model. In addition, the growth mechanism of the road network is developed to minimize the construction cost related to the population density and the Euclidean distance. The distribution complexities of the population density and the road network topology in the evolution process are then analysed. Compared with the real Beijing city in 2012, the suggested model can be used to describe the city evolution process.

1. Introduction

With the rapid development of urbanization, city evolution has been paid much attention in different fields. Generally, there have been two major categories in this issue, population expansion models and road network growth patterns.

Regarding city expansion, it is a very complex dynamic process influenced by many factors [1] such as population, roads, and economy. In city development process, the population density plays a pivotal role because it is regarded as an internal driving factor, and economy is considered as an external push of urban expansion. Generally, the population is used to show the size of the city. To capture the influence of population, Anas [2] proposed a dynamic model of city development based on the urban residential market. Dendrinos and Mullally [3,4] then presented dynamical differential equations and a predator–prey dynamic to describe the population dynamics and city evolution in particular. In 1999, Fujita et al. formulated a mathematical model to explain the hierarchical formation of cities from a "decentralized market process", which marked the arrival of the era of New Economic Geography [35]. In addition, Henderson and Venables [5] developed a dynamic model to analyse the problem of city formation and city size in an economy considering production, commuting and land rent, housing, subsidies and taxes. Recently, a cellular automata (CA) model, as an efficient simulation method, was adopted for the evolution of city expansion [6–8].

In addition to the population density, the growth of road networks is also a key factor in city evolution dynamics. Therefore, another issue of concern is how to describe the mechanism of road growth from the network generation algorithm [9],

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network topology structure [10–14], and the effects of social and economic activities [15]. Especially, Yamins et al. [16] presented a method to simulate the growth of urban transportation networks, which includes the selection of appropriate locations and a cost-based road building process. Li et al. [17,18] coupled the concepts of urban dynamics and transport development and discussed the results in detail based on a GIS technique.

Although there have been many efforts focused on the population density and road network growth, these models pay much attention to a single factor and ignore the relationship between road network growth and population density in city evolution. In fact, the population distribution and road network have a strong correlation in city evolution because population expansion can incur a significant travel requirement, which will stimulate the growth of the road network. Similarly, the growth of the road network will accelerate population expansion.

Some related works have also considered the urban land use and network coevolution process, the most important of which was proposed by Levinson and Xie [19,20]. They first presented the co-development model of land use and transportation network considering the network, population, and employment data; after that, they proposed some models including travel demand models, road investment models and accessibility and land use models. Subsequently, Barthélemy and Flammini [21] expanded the research of Levinson and described the effects of economic mechanisms on the evolution of the population density and the road network topology. Most recently, Wu et al. [22] proposed a new dynamics model based on the logistic equation to capture the dynamic characteristics of the coevolution process between the road surface and urban traffic structure. Zhao et al. [23], based on the research of Wu's group, further proposed an evolution model with the consideration of population distribution and the road network. However, the construction cost in the path building is scarcely considered in the work. After a comprehensive review of previous research, Li et al. [24] suggested an integrated coevolution general bi-level programming model of land use and traffic network design. They analysed and simulated the relationship between two types of economic agents—government investment decisions versus household and company locations—to show that average accessibility for employment and population increases in the evolution process. In general, this research provided us with brand new insight and further cognition of coevolution knowledge. However, these studies lack consideration of the effects of population diffusion, which is critical in the coevolution process. In addition, in the conventional context of urban planning, the CA method is widely used to simulate the complex urban growth process. Hence, the expansion of this research method is meaningful.

To fill the gap mentioned above, in this paper, we develop a coevolution model of population density and road network growth based on two previous approaches: the model of population diffusion by Li et al. [7] and the minimum construction cost method by Yamins et al. [16]. The contributions can be summarized as follows. First, we propose a new coevolution model with the consideration of population expansion and road network growth. In the model, we develop an approach to describe the people movement among the cells. The evolution of the population density prompts the establishment of roads to satisfy the travel demand, and the evolution of the road network is based on the minimum cost related to the distance and population [16]. In addition, we consider the random diffusion possibility when there is no road between two cells, which is ignored in the original model. Through this model, we can simulate the coevolution process of the urban population and road network. The mechanisms of urban expansion and the road network are uncovered, which are meaningful in urban and transportation planning.

The rest of this paper is organized as follows. The next section presents the coevolution model. The third section contains the simulations and analysis. The conclusion is given in the last section.

2. Coevolution model

In the following, we describe the notations used in the paper. Here, a city is divided into $I \times J$ square cells, where I is the number of rows, and J is the number of columns. Each cell contains the information on the population density and road network topology. Let $D[i, j]$ and $C[i, j]$ denote the population density and the status of the cell, respectively. Assume that each cell has three states, 0, 1 and 2, which represent sparsely populated, moderately populated and densely populated cells, respectively. Therefore, the status of cell $[i, j]$ can be presented by

$$C[i,j] = \begin{cases} 0, & \text{if} \quad D[i,j] < b_1 \\ 1, & \text{if} \quad b_1 \le D[i,j] < b_2 \\ 2, & \text{if} \quad D[i,j] \ge b_2 \end{cases}, \tag{1}$$

where b_1 and b_2 are the given critical value of the population density. In reality, these two parameters are difficult to determine because of the different city development principles. The transition of the status (that is, the transformed population density state) leads to the possibility of building a new road, which will be discussed in Section 2.3.

Let $R([i, j], [i', j'])$ be the accessibility flag describing a path existing between cells $[i, j]$ and $[i', j']$ or not, which can be written as

$$R([i,j],[i',j']) = \begin{cases} 1, & \text{at least one path between } [i,j] \text{ and } [i',j'] \\ 0, & \text{otherwise} \end{cases}. \tag{2}$$

In Eq. (2), $R(\cdot, \cdot)$ can be calculated through the shortest path algorithm according to the cell adjacency matrix Z. If the shortest path between $[i, j]$ and $[i', j']$ is less than ∞, let $R([i, j], [i', j']) = 1$.