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A spectral method for numerical modeling of radial microwave heating in cylindrical samples with temperature dependent dielectric properties

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ABSTRACT

In this paper, we develop a numerical model based on spectral methods for the simulation of heat transfer due to radial irradiation microwave applied to samples in cylindrical geometry. We solve the Maxwell's equations and the resulting electric field distribution is incorporated as a source term in the heat transfer equation. The model includes the temperature dependence of the dielectric properties. The numerical model is validated with experimental temperature data from literature.

1. Introduction

Microwave heating is one of the most important methods for heating materials. It is used for different processes such as baking, thawing, heating, cooking and drying [1–5]. Microwaves offer several advantages: high speed startup, less process time, internal heating, energy efficiency, making no pollution, which have made them a high demand technology in industrial and household applications.

Modeling of microwave heating involves coupling the models for microwave power absorption and temperature distribution inside the sample. Several simulation studies have modeled the heat generation due to microwaves by considering that the microwave power decreases exponentially as a function of penetration into the product. This simple approach, known as Lambert's law, is valid only for large sample dimensions and high loss dielectric materials [6]. In other case, a complete solution of the unsteady Maxwell's equations is required [7–9].

Simulation techniques have been extensively applied to model heat transfer due to microwaves. Among the computational methods available, Finite Element Method (FEM) and Finite Difference (FD) have been the most commonly used for solving microwave heating problems [10].

There are several studies on microwave heating of cylinders involving heating with radial irradiation using a finite element method [11–14].

The objective of this study is to provide a numerical method based on spectral methods for the simulation of heat transfer due to radial irradiation microwave in cylindrical geometry.

Spectral methods are usually the best tool for solving partial differential equations to high accuracy on a simple domain if data defining the problem are smooth. They can achieve more accuracy than a finite difference or a finite element method.

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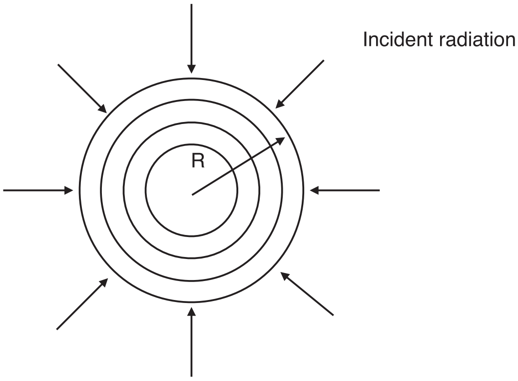

Fig. 1. Physical setup. Electromagnetic radiation incident normal to the surface.

They also demand less computer memory than the alternatives. From the three most commonly used spectral schemes the collocation method has been chosen. This is a numerical method often used in thermoconvective problems [15–18] which has been demonstrated to be an efficient and useful tool for solving numerically the partial differential equations that model those problems. In this method, the approximation of a field w is given by its projection w_N into a certain functional space of finite dimension (a space of polynomials). It is imposed that w_N verifies the discretization of the operator we want to solve in certain points (collocation points), chosen depending on the problem and the selection of the basis for the space in which w is approximated.

In the present work, we develop a numerical method based on a spectral collocation method to predict the temperature of a cylindrical sample heated radially by microwave.

2. Mathematical model formulation

2.1. Maxwell's power dissipation

The evaluation of the temperature distribution in any material subject to microwave irradiation depends on the knowledge of the electromagnetic field resulting from microwave power absorption. Several simulation studies model the heat generation due to microwaves by considering that the microwave power decreases exponentially as a function of penetration into the product. Such approach, known as Lambert's law, can be obtained through a series of simplifications applied to Maxwell's equations [19]. This is valid only for semi-infinite samples with dimensions much larger that the wave length.

A more rigorous procedure to evaluate the electromagnetic field distribution consists on solving Maxwell's equations:

$$\nabla \times \mathbf{H} = \mathbf{J} + \frac{\partial \mathbf{D}}{\partial t}, \tag{1}$$

$$\nabla \cdot \mathbf{D} = q \tag{2}$$

$$\nabla \cdot \mathbf{B} = \mathbf{0}, \tag{3}$$

where $\mathbf{E}$ is the electric field, $\mathbf{B}$ is the magnetic flux density, $\mathbf{H}$ is the magnetic field, $\mathbf{J}$ is the current density, $\mathbf{D}$ is the flux density and q the electric charge density. The constitutive relations are

$$\mathbf{J} = \sigma \mathbf{E}, \tag{4}$$

$$\mathbf{D} = \epsilon \mathbf{E}, \tag{5}$$

$$\mathbf{B} = \mu \mathbf{H}, \tag{6}$$

where σ is the electric conductivity, μ is the magnetic permeability and ϵ is the electric permittivity. With the help of the constitutive relations and assuming that the material magnetic permeability μ is approximated by its value in free space, considering electroneutral conditions of the material, and a one-dimensional analysis with the incident radiation assumed to be normal to the surface of the cylinder (Fig. 1), Maxwell's equations yield:

$$\frac{d^2E}{dr^2} + \frac{1}{r}\frac{dE}{dr} + k_1^2 E = 0, \quad \text{for } 0 < r < R, \tag{7}$$

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