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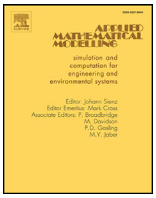

Imperfect interface effect for nano-composites accounting for fiber section shape under antiplane shear

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ABSTRACT

This paper investigates the elastic responses of fibrous nano-composites with imperfectly bonded interface under longitudinal shear. The proposed imperfect interface model is the shear lag (or the spring layer) model; the presented nano interfacial stress model is the Gurtin–Murdoch surface/interface model; and the three-phase confocal elliptical cylinder model is the geometry model accounting for the fiber section shape. By virtue of the complex variable method, a generalized self-consistent method is employed to derive the closed from solution of the effective antiplane shear modulus of the fibrous nano-composites with imperfect interface. Five existing solutions can be regarded as the limit form the present analytic expression. The influences of the interface elastic constant, the interfacial imperfection parameter, the size of the elliptic section fiber, the fiber section aspect ratio, the fiber volume fraction and the fiber elastic property on the effective antiplane shear modulus of the nano-composites are discussed. Particularly, numerical results demonstrate that the interfacial elastic imperfection will always cause a significant reduction in the effective antiplane shear modulus; and the fiber interface stress effect on the effective modulus of the fibrous nano-composites will weaken with the interfacial imperfection increases.

1. Introduction

Nano-composites are widely used in modern science and technology such as electromechanical systems, bioengineering, optics and photonics, etc. In the fabrication of advanced composite materials, one can improve mechanical properties of traditional composites with high elastic modulus and mechanical strength by using the nano-sized objects. For example, carbon nano-tubes have been used as nano reinforcements in the matrix systems of fiber reinforced plastics in order to improve out-of-plane properties, thus increasing the delamination resistance [1].

Before in fact, appears in the nano-meter material, the fiber reinforced composite materials once was an extremely useful concept. This paper is concerned with the longitudinal shear prediction. Here, a brief review of the modulus prediction associated with longitudinal shear for fibrous composite is given as follows. In 1963, Hashin and Shtrikman [2] established the bounds of the shear modulus for a mixture formed with two isotropic phases firmly bonded together via the variation principle. Afterward, Hashin and Rosen [3] made an estimation of the effective shear modulus for the composite materials reinforced by parallel hollow circular fibers for the hexagonal array and the random array. They pointed out the random-array results are much to be preferred because of their much simpler form and the coincidence of the bounds for effective

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Nomenclature

λ	fiber volume fraction
a_1, b_1	the semimajor axes and semiminor axes of fiber S_1
a_2, b_2	the semimajor axes and semiminor axes of matrix S_2
τ_{yz}^{∞}	far-field antiplane shear stress
u_x, u_y, u_z	the displacement components in the x, y, and z directions
$w(x, y)$	the displacement function in the z direction
τ_{xz}, τ_{yz}	the stresses related to w in Cartesian coordinate
τ_{zr}, $\tau_{z\theta}$	the stresses related to w in polar coordinates
$\tau_{z\theta}^{0}$	the circumferential stress on the imperfect interface
τ_{zr}^{0}	the radial stress on the imperfect interface
γ_{zx},γ_{zy}	the strains related to w
T_τ	the resultant of the stress
$\zeta=\xi+i\eta=le^{i\phi}$	the ζ-plane
$\Omega(\zeta)$	mapping function
ρ_0,ρ_1,ρ_2	the radius of concentric circles in ζ-plane
λ^S,μ^S	the surface Lame constants
τ^0	the residual surface stress
μ^*	the joint expression of interfacial elastic constant and residual tension
$\tau_{z\theta}^{0}$	the circumferential stress on the imperfect interface
τ_{zr}^{0}	the radial stress on the imperfect interface
$\varepsilon_{z\theta}^{0}$	the interfacial strain components on the interface
α	a non-negative parameter
μ_A	interface constant
$F_1(\xi)$, $F_2(\xi)$ and $F_3(\xi)$	analytical functions
a^*,a_k	complex constants to be determined
A_1, B_1, B_{-1}, C_1 and C_{-1}	real constants
$\tilde{\gamma}_{xz}$, $\tilde{\gamma}_{yz}$	average value of strain
$\tilde{\tau}_{xz}$, $\tilde{\tau}_{yz}$	average value of stress
G_E	the effective modulus of the nano composites
χ	the dimensionless effective modulus
γ	the section aspect ratio of the elliptical fiber
δ	the dimensionless parameter for interfacial imperfection
β	the dimensionless parameter for interfacial stress

shear modulus. The self-consistent method most often employed in studies on multiple-phase composites has been proposed for aggregates of crystals by Hershey [4] and Kröner [5]. This self-consistent method has been developed to estimate the macroscopic elastic moduli of two-phase composites by Hill in 1965 [6]. Chen used the theory of elasticity and derived the composite elastic moduli of parallel fibers also in a hexagonal packing [7] and a square array [8]. Adams and Doner [9] formulated the numerical solution for a unidirectional fiber-reinforced composite material subjected to a shear loading using a finite difference representation and a numerical relaxation procedure. Their results [9] show that the square array gives higher predicted moduli than the hexagonal one for the same fiber volume content, and the predicted values vary considerably at high fiber concentrations. Based on a generalized Einstein coefficient and a function related to the maximum volumetric peaking fraction of the filler phase, Nielsen [10] proposed a generalized equation for the relative elastic moduli of composite materials which fit better with the computer calculations of Adams and Doner [9] at high fiber concentrations. Symm [11] given the improved Heaton's prediction [12] of the modulus associated with longitudinal shear of a unidirectional fibrous composite for square array and hexagonal array. Symm [11] observed that for low fiber concentration the values of longitudinal shear modulus prediction derived for hexagonal and square arrays are identical; and these values agree exactly with values obtained from a theoretical formula of Hashin and Rosen [3] for a "random array" with transverse isotropy. It is well known that the filaments in unidirectional fiber reinforced composites are non-uniformly distributed and have varying radii and elastic properties. Taking into account random spacing between fibers, random variation of fiber radii, and the variation in shear modulus from fiber to fiber, Sendeckyj [13] obtained the exact analytical solution for a case of longitudinal shear loading of a unidirectional fiber reinforced composite. His solutions give results which are in close agreement with some numerical solutions of Chen [8], Adams and Doner [9]. For the three-dimensional random orientational fiber composites, the predicted formulae of the effective isotropic properties derived by Christensen and Waals [14]. Christensen and Lo [15] employed inclusion/matrix/composite three phases model to solve the effective shear modulus of two types of composite material models. In the above works, the fiber/matrix model of a combined circular cylinder is occasionally applied to predict the effective moduli of fiber reinforced composite materials. The moduli prediction for composite material with

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