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Modelling the dynamic response of a micro-cantilever excited at its base by an arbitrary thermal input using Laplace transformation

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ABSTRACT

An analytical model for predicting the time-history response of a cantilever beam to arbitrary time-dependent thermal actuation is elaborated in this paper. Base excitation is investigated as a practical method for thermally exciting the micro-cantilever. The beam is considered to be mounted on a layer of material (actuator) that is thermally excited (e.g., by electric current). Thermal expansion/contraction of the base causes the micro-cantilever to vibrate. One-dimensional heat conduction equation is solved for the actuator, along with the Euler–Bernoulli continuous beam equation for the micro-cantilever. An arbitrary time dependant body heat generation is applied on the actuator as the excitation function for the latter equations. Laplace transformation is applied to tackle the time dependency of the partial differential equations. After solving the coupled ordinary differential equation, two methods based on Gaver–Stehfest algorithm and direct numerical integration are considered for the inverse transformation and discussion regarding results and procedures are presented. Moreover, a case study of a thermally actuated resonator with periodic input signal is investigated and conclusions on the practical design and implementation are demonstrated.

1. Introduction

Micro-electro-mechanical systems (also known as MEMS) are found in a wide variety of applications in industry. Applications span from aerospace and defence to automobile and home appliances [1]. The advantage of MEMS in compact design as well as their low energy consumption and high accuracy in sensing has promoted them as a valuable technology for a variety of applications. On the other hand, considering the fact that MEMS devices are relatively new to the engineering world, research and development are still required in different aspects of this field.

MEMS devices are used for sensing and/or actuation. In general, some of the negligible factors in macro-scale are major phenomenon in micro and nano scales. For instance, the extremely small sizes of micro-level features compared to macro-scale components and parts reduces the thermal time constants of MEMS parts to substantially small values [1], which allows thermal cooling and heating cycles of up to a few GHz [2,3]. Moreover, due to their small sizes, micro-level components are reported to have relatively high natural frequencies [4] for their vibrational modes. Combination of small thermal time-constant and high natural frequencies allows for the implementation of (electro-) thermally actuated MEMS devices

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Nomenclature

b	the dominant length scale in hydrodynamic force equation
C_p	specific heat capacity of actuator
E	modulus of elasticity for the beam material
E_{base}	modulus of elasticity for the actuator material
f	frequency
F_1	interaction force between actuator and beam
h	height of the actuator
i	$\sqrt{-1}$
I	second moment of inertia of the beam cross section
K_{base}	stiffness of the actuator (base)
l	length of the actuator layer
m	mass per unit length of the beam
$p(x, t)$	distributed load along the length of the beam at location x and time t
P	fluid pressure
P_{in}	input power
$q(t)$	heat generated per unit volume in the actuator (time-dependent)
t	time
u_1	displacement at the top of the actuator
$u(x, t)$	displacement at location x along the beam at time t
$\vec{v}$	fluid velocity vector
V_{act}	volume of actuator
w	out of plane thickness of system (actuator and beam)
z	spatial coordinate along the actuator
α	thermal expansion coefficient of the actuator
η_f	fluid viscosity
$\theta(z, t)$	temperature at location z and time t (relative to reference)
κ	thermal conductivity of actuator
ρ	mass density of actuator
ρ_f	mass density of the fluid

for various applications [1,4]. Besides the innovative designs for achieving large deflections using thermal actuation (such as cold-hot arm [5], out-of-plane buckling [6,7], and using large leverages [8]), the bi-morph effect based on the difference in thermal expansion coefficient of two materials is the most common method for thermal excitation of MEMS devices [1,9]. However, bi-morph effect can result in high stresses at the interface between the two materials that might result in crack initiation and propagation.

Micro-cantilever configuration is amongst the most common architectures of implementing MEMS for various applications [1,4]. Micro-cantilevers are in use for numerous purposes including but not limited to energy harvesting [10–12], optical micro-mirrors used in image acquisition [6,13] and micro-scanners [9], chemical [14] and bio-sensors [15] etc. New applications for MEMS in general and micro-cantilevers in particular are introduced every year.

The focus of this paper is on the results of research conducted on analytical modelling of thermally actuated micro-cantilevers that are vibrated using base excitation. Applying base displacement is a well-known technique for inducing vibration in long and thin objects (such as a cantilever beam) [16]. Moreover, applying a periodic (cyclic) load or displacement at this base with a frequency that corresponds to the natural frequency of the micro-cantilever results in resonance in the system. Maintaining the source of excitation for long enough duration of time results in quite large amplitudes of displacement (theoretically out of bond vibrations in absence of energy loss) [16]. In fact, the effect of resonance has been used in practice in design of MEMS in order to enhance performance. In other words, if the micro-cantilever is designed to have a natural frequency (preferably the first natural frequency) that matches the provided excitation frequency, larger amplitudes of displacements (deformations in general) can be achieved compared to non-resonance case.

This paper first looks into the heat conduction equation [17], the one-dimensional (1D) form of this equation is solved taking into account the internal heat generation. The temperature distribution inside the actuator is formulated as a function of body heat generation input. Then, the boundary value problem relating the continuous beam dynamics to the time-dependant thermal expansion at the actuator is expanded. Time-dependency of these equations has been resolved via Laplace transformation [18]. This leads to ordinary differential equations that can be easily solved. However, the major challenge to tackle in this case is the inverse Laplace transformation. Due to complexity of the resultant equations, this cannot be easily solved through direct inverse Laplace. Therefore, two alternative methods four numerical Laplace inverse are presented and their effectiveness for this analysis is discussed here.