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A circular inhomogeneity with a mixed-type imperfect interface in anti-plane shear

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ABSTRACT

We present a rigorous study of the problem associated with a circular inhomogeneity embedded in an infinite matrix subjected to anti-plane shear deformations. The inhomogeneity and the matrix are each endowed with separate and distinct surface elasticities and are bonded together through a soft spring-type imperfect interphase layer. This combination is referred to in the literature as a 'mixed-type imperfect interface' due to the fact that the soft interphase layer (described by the spring model) is bounded by two stiff interfaces arising from the separate surface elasticities of the inhomogeneity and the matrix. The entire composite is subjected to remote shear stresses and we allow for the presence of a screw dislocation in either the inhomogeneity or the matrix. The corresponding boundary value problem is reduced to two coupled second-order differential equations for the two analytic functions defined in the two phases (as well as their analytical continuations) leading to solutions in either series or closed-form. The analysis indicates that the stress field in the composite and the image force acting on the screw dislocation can be described completely in terms of three size-dependent parameters and a size-independent mismatch parameter. Interestingly, in the absence of the screw dislocation, the size-dependent stress field inside the inhomogeneity is uniform. Several numerical examples are presented to demonstrate the solution for a screw dislocation located inside the matrix. The results show that it is permissible for the dislocation to have multiple equilibrium positions.

1. Introduction

The micro-mechanical analysis of composite materials is often concerned with finding the elastic fields of a single inhomogeneity embedded within another elastic medium (for example, in the analysis of fiber-reinforced composites). In order to reduce the complexity of the analysis and to arrive at a tractable problem, much of the initial progress in this area was made using the classical assumption that the inhomogeneity was 'perfectly bonded' to the surrounding elastic matrix. Specifically, this meant that the mathematical model describing deformations of the composite material was developed under the assumption that displacements and surface tractions are continuous across the inhomogeneity-matrix boundary, the

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so-called ‘perfect bonding condition’. The perfect bonding condition, however, is actually an idealization of a more complex situation. In fact experimental evidence is quite clear in the conclusion that load transfer between the inhomogeneity and the matrix depends significantly on the properties of the so-called interphase layer (a region between the inhomogeneity and the surrounding matrix). Consequently, the incorporation of the contributions of an interphase layer into any micro-mechanical model is essential in describing the physical and mechanical behavior of the composite material. The concepts known as imperfect bonding and imperfect interface have been developed in order to take into account these contributions.

Over the last several decades, two types of imperfect interface models have arisen most frequently in the literature. The first is based on the so-called soft or spring-type imperfect interface [1–3]. This model assumes that tractions are continuous but displacements are discontinuous across the interface. More precisely, jumps in the displacement components are proportional, in terms of 'spring-factor-type' interface parameters, to their respective interface traction components. The soft interface model can be used to describe various kinds of damage (e.g., debonding, sliding and/or micro-cracking) occurring along the interfacial region between two neighboring materials (see, for example, [4,5]). The second model is based on a so-called stiff imperfect interface [1]. This model is based on the assumption that displacements are continuous but tractions are discontinuous across the interface. More specifically, the corresponding jumps in tractions are proportional to certain surface differential operators of the displacements [6]. Interestingly, the isotropic version of the stiff imperfect interface model is essentially equivalent to the mathematical description of the Gurtin–Murdoch surface elasticity without the effects of surface tension [7–11]. The Gurtin–Murdoch surface model has been incorporated into the micromechanics analysis of nano-sized inhomogeneities by various authors [12–18].

In this paper, we consider the more practical (albeit more challenging) ‘hybrid scenario’ in which separate surface elasticity is incorporated simultaneously onto the surface of the inhomogeneity and onto the adjoining surface of the matrix with the two phases bonded together through a soft spring-type imperfect interface layer. The presence of the surface elasticities again (as in the aforementioned works of Sharma et al. [15], Duan et al. [13], Yang [18], Chen et al. [12], Tian and Rajapakse [16], Luo and Xiao [14] and Wang and Schiavone [17]) allows our model of deformation to accommodate inhomogeneities which are essentially ‘nano-sized’. This is based on the well-established conclusion in the literature that surface effects are known to contribute significantly to models of deformation at the nanoscale. We refer to the situation in which soft and stiff interfaces co-exist in this fashion as a mixed-type imperfect interface. Recently, we have used an analogous mixed-type imperfect interface model to obtain a closed-form solution to the problem of plane deformations of a circular elastic inhomogeneity embedded in an infinite matrix subjected to uniform remote in-plane stresses [19]. That particular solution showed the versatility and practicality of the mixed-type interface model in that it also allowed for a simple and straightforward method for the design of both harmonic and neutral elastic inhomogeneities, each of which plays a significant role in the manufacture of composite materials, for example, in the design of implants for biomechanical purposes (see [19] and the references therein). The present paper extends the analysis in [19] by applying the idea of a mixed-type imperfect interface to examine another important problem in materials science: the influence of a screw dislocation on the mechanical behavior of a composite material subjected to anti-plane shear loadings. In materials science, it is well-known that the presence of dislocations strongly influences many of the properties of materials. It is commonly observed that dislocations in structural materials tend to be distributed in organized patterns. Accordingly, a basic problem in dislocation theory is to determine the equilibrium arrangement of dislocations under their mutual interactions and under the action of an applied stress field. The pursuit of equilibrium positions for the dislocation is therefore one of the most important problems in composite mechanics from both theoretical and practical perspectives. For example, a stable equilibrium position for the dislocation means that the dislocation will be lodged in that position (see, for example, [20,21]).

Recently, Wang and Schiavone [22] used the Stroh formalism to obtain the Green's functions for an anisotropic bimaterial with a planar mixed-type imperfect interface subjected to a line force and a line dislocation. In the present work, we analyze the anti-plane shear deformations of a nano-sized circular inhomogeneity with a mixed-type imperfect interface embedded in an infinite matrix subjected to various loadings including uniform anti-plane stresses at infinity and a screw dislocation applied in the matrix or in the inhomogeneity. It is observed that in the absence of any screw dislocation, the stress state inside the circular inhomogeneity with a mixed-type imperfect interface remains uniform although size-dependent. The numerical results indicate that the co-existence of the soft and stiff interfaces makes it permissible for the screw dislocation to have multiple equilibrium positions in the matrix.

2. Basic formulation

2.1. The bulk elasticity

In what follows, unless otherwise stated, Latin indices i, j, k take the values 1,2,3 and we sum over repeated indices. In a Cartesian coordinate system $\{x_i\}$, the equations of equilibrium and the stress–strain relations describing the deformation of a linearly elastic, homogeneous and isotropic bulk solid are:

$$\sigma_{ij,j} = 0, \quad \sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\varepsilon_{kk}\delta_{ij}, \quad \varepsilon_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \tag{1}$$

where λ and μ are Lame constants, σ_{ij} and ε_{ij} are, respectively, the components of the stress and strain tensors in the bulk material, u_i is the ith component of the displacement vector $\mathbf{u}$ and δ_{ij} is the Kronecker delta.