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Combinatorial approach to exactly solving discrete and hybrid berth allocation problem

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ABSTRACT

This paper presents an exact combinatorial algorithm for solving the *Discrete Berth Allocation Problem (DBAP)* and the *Hybrid Berth Allocation Problem (HBAP)* with fixed handling times of vessels based on the original algorithm for solving combinatorial problems called *Sedimentation Algorithm*. We address the issues of DBAP and HBAP according to the Rashidi and Tsang model. To the best of our knowledge, the proposed algorithm is the first exact combinatorial algorithm for solving the general DBAP and HBAP based on Rashidi and Tsang model. Computational results prove the superiority of the proposed algorithms compared with the exact solvers based on the Mixed Integer Programming (MIP) models. Efficient C implementation enabled us to solve instances with up to 65 vessels. This resolves most of the real life problems, even in large ports.

1. Introduction

The *Berth Allocation Problem* (BAP) consists of allocating berths to a set of vessels that need to be served within a given time horizon in a container port. Vessels are, among other information, represented by a set of data that includes the expected time of arrival, size, projected handling time, preferred berth in the port, and penalties. BAP can be defined as follows: for each vessel in the set, the berth index and the time interval are allocated in the manner that the given objective function is minimized. In Lim [1], BAP was proven to be a NP-hard problem.

BAPs can be classified as discrete, continuous or hybrid, see [2]. In DBAP quay is partitioned into a number of units, called berths, and each berth can serve one vessel at a time. Each vessel, on the other hand, occupies exactly one berth. Time is also partitioned into discrete units, which allows the usage of integer arithmetic for the calculation of the objective function value. HBAP is similar to DBAP with the exception that a larger vessel can occupy several successive berths while smaller vessels can share one berth. Conversely, in the continuous BAP there is no partitioning of the quay, i.e. vessels are allowed to take arbitrary position within the boundaries of the quay. Another possible classification distinguishes static and dynamic BAPs. In the static BAP, it is assumed that vessels arrival times impose soft constraint on the berthing times. The vessels already wait at the port and can berth immediately or the vessel can be speeded up in order to meet berthing time earlier than the expected arrival time. In the dynamic BAP fixed arrival times are given for the vessels, hence, vessels cannot berth before expected arrival time. A detailed BAP classification can be found in [2,3].

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In recent literature, exact approaches addressing BAP are rare, the majority of studies use heuristic or meta-heuristic methods to obtain suboptimal solutions of BAP. According to the recent surveys of BAP by Bierwirth and Meisel [2,4] exact methods are applied in 24% of approaches, while the rest of 76% approaches belongs to the heuristic and meta-heuristic methods.

An exact method for solving BAP can be found in Vacca et al. [5]. The authors proposed an exact algorithm for solving the *Tactical Berth Allocation Problem* (TBAP) defined by Giallombardo et al. [6]. The model for TBAP is based on an exponential number of variables, and it is solved via column generation. To obtain an integer solution, a branch-and-price scheme was applied along with several accelerating techniques specifically designed for solving TBAP.

Although BAP in container port is different from BAP in bulk port we list two works addressing BAP in bulk port. In Umang et al. [7] two exact methods, based on mixed integer programing and generalized set partitioning, and one heuristic methods to solve dynamic hybrid BAP in bulk ports are proposed. Robenek et al. [8] propose branch-and-price exact solution algorithm.

Heuristic and meta-heuristic methods for solving BAP are far more common. The following short review does not cover this topic completely. Its purpose is just to illustrate the variety of approaches.

Both static and dynamic discrete BAPs were examined by Imai et al. [9]. In both problem variants, the assignment and sequencing of vessels to berths was determined by minimizing the vessels' waiting and handling times. A Lagrangian relaxation-based heuristic was used to solve the problem. A similar approach, with a stronger Lagrangian relaxation because of the different formulation used, was applied by Monaco and Samara [10] for the dynamic version of DBAP. Cordeau et al. [11] modeled DBAP as a Multi-Depot Vehicle Routing Problem with Time Windows and applied the Tabu Search meta-heuristic to find good sub-optimal solutions for the problem. A similar approach was adopted by Mauri et al. [12] to solve DBAP. The set partition approach was used by Cristensen and Holst [13] to solve DBAP. Zhen et al. [14] applied a Simulated Annealing meta-heuristic, whereas de Oliveira et al. [15] applied Clustering Search method using Simulated Annealing for solutions generation. Lee and Chen [16] and Hansen et al. [17] used a Variable Neighborhood Search for the same variant of the problem. Genetic Algorithms were applied to several variants of DBAP by Imai et al. [18], Han et al. [19], Zhou et al. [20], and Nishimura et al. [21]. Iterated Greedy Heuristic for solving DBAP was used by Lin et al. [22]. Lalla-Ruiz and Voß [23] employ Partial Optimization Metaheuristic Under Special Intensification Condition Metaheuristic (POPMUSIC) for solving DBAP.

HBAP with fixed handling times was examined by Chen and Hsieh [24] using the MIP problem formulation. In order to solve HBAP Moorthly and Teo [25] used a precedence graph representation which is analyzed using the Project Evaluation Review Technique. Dai et al. [26] proposed Simulated Annealing algorithm for solving the same version of HBAP. Bee Colony Optimization was applied by Kovač [27] for solving the Minimum Cost Hybrid BAP with fixed handling times of vessels.

The HBAP formulations with position dependent vessel handling times are studied in several papers. Imai et al. [28] investigate indented berths HBAP. Also, Imai et al. [29] developed Genetic Algorithm for the berth allocation of the mega-ships served from two sides. Cordeau et al. [11] obtain HBAP from DBAP. The works of Nishimura et al. [21], Cheong et al. [30] and Hoffarth and Voß [31] include the vessels' draft into HBAP. The same work proposed a heuristic for solving HBAP.

In this paper the discrete BAP (DBAP) and the hybrid BAP (HBAP) are considered. We present an original exact approach for solving DBAP and HBAP implemented in two variants. The first variant, named *Sedimentation Algorithm* (SEDA), is a general combinatorial optimization algorithm adopted for solving BAP. SEDA is an exact solver and it works on the combinatorial branch and bound principles. The second variant differs from the first one because it uses a heuristic in the pre-processing phase to reduce the search space for SEDA. We name this algorithm the *Sedimentation Algorithm with an Estimation & Rearrangement Heuristic* (SEDA+ERH). To the best of our knowledge, these are the first exact combinatorial algorithms for solving the general DBAP and HBAP based on Rashidi and Tsang [32] model. Estimations of the complexity of the algorithms are given. Numerical experiments are conducted on four sets of test examples involving 5, 8 or 13 berths with one or two-week time horizon. SEDA+ERH enable us to find, in a very short CPU times, the optimal solution for the larger problem instances with up to 60 vessels to be scheduled during the time horizon of one or two weeks. In addition, we compare our combinatorial approach (realized through the implementation of SEDA and SEDA+ERH) against commercial MIP based exact solver CPLEX. Our computational results clearly prove the superiority of the proposed combinatorial algorithms.

The rest of this paper is organized as follows. At the beginning, SEDA and SEDA+ERH are described in Section 2. Then BAP notation and problem formulation are introduced in Section 3. BAP modeling for SEDA and the complexity of SEDA for solving BAP are described in Section 4. Computational results are presented in Section 5. Finally, Section 6 contains concluding remarks and directions for future research.

2. The sedimentation algorithm

Sedimentation Algorithm (SEDA) is a general combinatorial optimization algorithm introduced for the first time by the authors in Kordić et al. [33] for solving the Berth Allocation Problem (BAP) at *International Association of Maritime Economists Conference, IAME, Taipei, 2012*. Since the proceedings of this conference is not accessible to wider scientific community here we present in detail SEDA and SEDA+ERH. The initial version of SEDA and its preliminary computational results presented at [33] are significantly improved and presented here.