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A dual beam model for geosynthetic-reinforced granular fill on an elastic foundation

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ABSTRACT

In this study, a new dual beam model was proposed for a geosynthetic-reinforced granular fill with an upper pavement. This dual beam model was subjected to a uniform surcharge loading and resting on an elastic foundation which was simulated by a Pasternak model. The upper pavement was modeled by an Euler–Bernoulli beam while the geosynthetic reinforced granular fill was simulated by a reinforced Timoshenko beam. The explicit derivation process for the behavior of this dual beam-foundation system was presented and an exact solution was obtained. A two-dimensional finite element analysis and a Pasternak model for simulating the granular fill were carried out to validate the reliability of the proposed dual beam model. A parametric analysis was put forward to investigate the behavior of this dual beam-foundation system. It was found that the length of the pavement structure and vertical uniform loading, the stiffness and shear modulus of the foundation soil had significant influences on the behavior of the dual beam-foundation system.

1. Introduction

Structures, such as pavements and rail tracks, built on continuous foundation soil are commonly idealized as beam-soil systems to study the behavior of foundation soil and stresses within the structures [1–6]. For the foundation soil, Winkler [1] proposed a spring analogy method named Winkler model to simulate the foundation soil. Pasternak [2] proposed a two-parameter model which can consider the transverse shear deformation to make up the deficiency of the Winker model and it is widely used by many researchers [7–9]. When it comes to a long and thin pavement structure, the Euler–Bernoulli beam theory is suitable for the computation of flexural deformation. To further consider the shear deformation effects in a thick beam, the Timoshenko beam theory is more suitable than the Euler–Bernoulli beam theory. The Timoshenko beam theory is also referred as first-order shear deformation theory (FSDT) which assumes the transverse shear strain is constant through the beam thickness, i.e. it assumes the cross sections remain plane and undistorted after deformation. Recently, some scholars proposed new FSDTs for laminated composite plates and beams [10–11]. Higher-order shear deformation

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theories (HSDTs) have also attracted much attentions as they can provide more accurate predictions for static and dynamic analysis of beams and plates [12–17]. Nevertheless, the Timoshenko beam theory is the simplest solution for the analysis of a granular fill layer whose thickness cannot be neglected [6,18].

Granular fill is good in compression but weak in tension. Engineers applied geosynthetics in practice to complement this weakness of granular fill. A number of scholars have used different models to simulate the behavior of reinforced granular fill [19–22]. Yin [23] developed a nonlinear constitutive model for granular fill to represent the nonlinear shear stress-shear strain behavior of granular fill reinforced with geosynthetics, and then used a finite difference scheme to solve it because of the complex mathematical derivations. Zhan and Yin [24] studied the soil-geosynthetic interaction using a two-dimensional analytical model where vertical interaction was simulated using Winkler springs and horizontal interaction was simulated using shear springs. Deb et al. [25] proposed a model to simulate the behavior of granular foundation beds reinforced with several layers of geosynthetics by using an iterative finite difference scheme. Deb et al. [26] also investigated the behavior of a multi-layer geosynthetic reinforced granular bed using the FLAC program. Maheshwari and Kashyap [27] presented the stochastic design of beams on reinforced random earth beds in deterministic mode. Rajesh et al. [28] studied the behavior of geosynthetic reinforced railway track system that rested on soft clay subgrade and used the finite difference scheme to obtain the numerical solutions. These researchers concentrated on the interaction between granular fill and the geosynthetics, and because the mechanisms are complex, these simulation models were solved mostly by numerical methods or were based on numerical software. Some other researchers idealized reinforced geosynthetics as beams with smooth surfaces and studied the interaction between soil, structure, and reinforcement [29–31]. These models can simulate geosynthetics such as geocell which possesses bending stiffness but when it comes to supple geosynthetics that only behaves in a tensile manner these models are not good at simulating the behavior of geosynthetic reinforced granular fill. Yin concentrated on the tensile property of geosynthetics and applied the reinforced Timoshenko beam model to simulate the mechanism of geosynthetic reinforced granular fill subjected to a point load [32] and any kind of loading [33]. Shukla and Yin [34] assumed geosynthetic reinforced granular fill as a reinforced Timoshenko beam and investigated the time dependent settlement due to the consolidation of saturated soft foundation. All these studies concentrated on the reinforced granular fill without an upper pavement structure.

In this study a rigorous analytical solution for predicting the behavior of geosynthetic reinforced granular fill with an upper pavement structure was proposed. A dual beam model (Euler–Bernoulli beam and Timoshenko beam) was used to simulate a structure-reinforced granular fill system. The long thin pavement structure was assumed as an Euler–Bernoulli beam. The granular fill embedded within the geosynthetics was simulated as a reinforced Timoshenko beam because it can illustrate the deformations of shear and flexure simultaneously. The underlying foundation soil was simulated by a Pasternak model. In addition to the proposed exact solution, a parametric analysis was carried out to study how the properties of the foundation soil affected the settlement and bending moment of the upper pavement structure, as well as the deformation and force in the reinforced geosynthetic layer.

2. Mathematical model and governing equations

In the mathematical model, both the extra loading and geometry were symmetric to the central line and Fig. 1 shows half of the model. The $2L_1$ long pavement structure was simulated by an Euler–Bernoulli beam because the pavement structure is relatively thin. The geosynthetic reinforced granular fill was simulated by a reinforced Timoshenko beam. The thickness of the granular fill layer was h and the length was infinite in the x (horizontal) direction. The length of the geosynthetic reinforcement was $2L_2$ and the thickness can be neglected compared to that of the granular fill layer. The Pasternak model

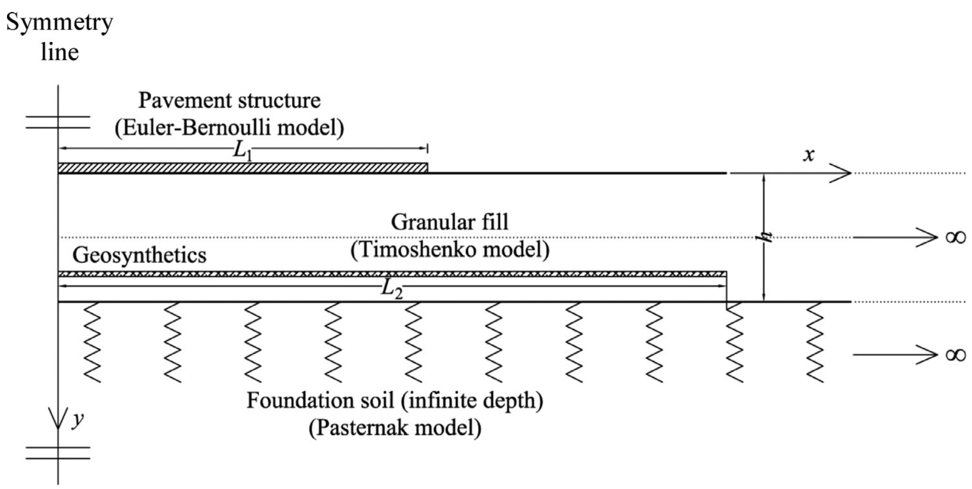

Fig. 1. Mathematical model of geosynthetic-reinforced granular fill with an upper pavement structure supported by foundation soil.