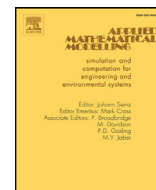




Contents lists available at ScienceDirect

Applied Mathematical Modelling

journal homepage: www.elsevier.com/locate/apmModelling effects of treatment at home on tuberculosis transmission dynamics[☆]Hai-Feng Huo^{*}, Ming-Xuan Zou

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ARTICLE INFO

Article history:

Received 25 January 2015

Revised 16 April 2016

Accepted 16 June 2016

Available online xxx

MSC:

92D30

Keywords:

Treatment

Tuberculosis

Equilibrium

Global Stability

ABSTRACT

A tuberculosis model with two kinds of treatment, that is, treatment at home and treatment in hospital, is constructed. Mathematical analyses show that the dynamics of model are determined by the basic reproduction number R_0 . If $R_0 < 1$, then the disease free equilibrium is globally asymptotically stable. If $R_0 > 1$, the endemic equilibrium is globally asymptotically stable when patients who are not cured do not transfer from hospital to home. Numerical simulations are also given to support our theoretical results. Our results show that the treatment at home has great negative influence on the spread of the tuberculosis.

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1. Introduction

Tuberculosis (TB) is a widespread infectious disease usually caused by *Mycobacterium tuberculosis*. Tuberculosis frequently attacks the lungs, but can also affect other parts of the body. TB is often described as a slow disease since it has long latency period distribution and short infectious period distribution. People who are latently infected with tuberculosis do not have symptoms [1]. Most latently infected individuals do not become infectious (active TB), about 5–10% of the latently infected individuals eventually progresses to active TB; that is, about 90–95% remain latently infected. Meanwhile most secondary infections are a consequence of long term and continuous contacts with a primary case or exogenous reinfection [2,3]. However, the risk of developing TB as a consequence of exogenous reinfection is lower than that of developing the primary case for most age groups [4].

According to the report of WHO [5], from 122 countries that account for 95% of reported TB cases, funding for TB prevention, diagnosis and treatment reached US\$6.3 billion in 2014, almost double the level of 2006. Compared with the latest global estimates of resource requirements for a full response to the TB epidemic, this leaves a gap of around US\$2 billion per year. The cost per patient treated for drug-susceptible TB in 2013 was in the range of US\$ 100–500 in most countries. The cost per patient treated for MDR-TB was most often in the range US\$ 5000–10,000. The average varied from US\$ 9235 in low-income countries to US\$ 48,553 in upper middle income countries.

Currently, the income of public hospitals in China consists of three parts, namely the drug income, medical technology services revenue and financial assistance. In 2004, the government funding accounted for only 7% of total hospital revenue,

[☆] This work was partially supported by the NNSF of China (11461041), the NSF of Gansu Province of China (148RJZ024).

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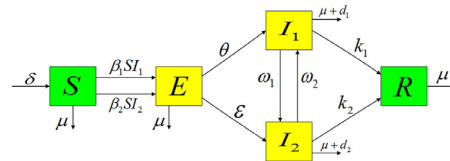


Fig. 1. Transfer diagram of the model (2.1).

the rest rely on their own income. So, the patients are not only required to afford medical costs, but also burden the hospital salaries, bonuses, buying equipment, and building the ward. In order to make up for inadequate compensation, the government allows hospitals to add a drug component 15%, which means that the higher the drug purchase price, the higher the income [6,7]. Thus, the personal burden proportion of the total cost of the national health is too high. According to report of the ministry of health, the expenses of hospital outpatient and hospitalization are per capita average annual growth of 13% and 11%, respectively, which are much higher than the growth rate of per capita income [7,8].

Mathematical models can provide useful tools to analyze the spread and control of Tuberculosis and other infectious disease, Bowong and Tewa [9] introduced a SEI type of tuberculosis model with a general contact rate and derived the stability of equilibria by the use of Lyapunov stability theory and LaSalle's invariant set theorem. Huo et al. [10] presented a two-strain tuberculosis model with general contact rate which allows tuberculosis patients with the drug sensitive of strain *Mycobacterium tuberculosis* to be treated and gave a detailed qualitative analysis about positivity, boundedness, existence, uniqueness and global stability of the equilibria of the model. Huo and Feng [11] constructed an HIV/AIDS epidemic model with different latent stages and treatment. The model allows for the latent individuals to have the slow and fast latent compartments. Other mathematical models for TB or other infectious disease have been formulated and studied (see [12–17] and the references cited therein).

Due to the emergence of multi-resistance, the treatment of TB is difficult. Drug-resistant TB is one of serious public health issues in many developing countries, since its treatment is longer and requires more expensive drugs. But the treatment is not completely free in many developing countries. Some TB patients cannot fully afford expenses of treatment. Thus, some TB patients are treated at their homes in order to save money. Some patients who were not cured, choose to discharge and go back to treat at home.

Motivated by the above, we will construct a new tuberculosis model with two kinds of treatment. Symptomatic infectious individuals in our model are divided into the following two compartments: symptomatic infectious individuals treated at home and symptomatic infectious individuals treated in hospital. We lay emphasis on the modeling effects of treatment at home in this paper. Comparing with our previous work [11], the main distinctions are, a Tuberculosis model is considered in this paper but an HIV/AIDS epidemic model is considered in [11]. The symptomatic infectious individuals are divided into two compartments in this paper while the latently infected individuals are divided into fast and slow compartments in [11].

The paper is organized as follows. In Section 2, a tuberculosis model with two kinds of treatment is formulated. In Section 3, the threshold conditions for the existence and uniqueness of equilibria are derived and the global stability of equilibria are also proved. In Section 4, numerical simulations are carried out to confirm our results. We give some discuss in the last section.

2. Model construction

The total population size is $N(t)$, which is divided into five compartments, namely, susceptible individuals (S), undetected non-symptomatic (latent) carriers (E), symptomatic infectious individuals treated at home (I_1), symptomatic infectious individuals treated in the hospital (I_2), and recovered (people who form calcified points in the lungs) (R). Where,

$$N(t) = S(t) + E(t) + I_1(t) + I_2(t) + R(t).$$

The model is shown by the following transfer diagram (see Fig. 1).

The process depicted in Fig. 1 is modeled by the following system of ordinary differential equations:

$$\begin{aligned}\dot{S} &= \delta - (\beta_1 I_1 + \beta_2 I_2)S - \mu S, \\ \dot{E} &= (\beta_1 I_1 + \beta_2 I_2)S - (\theta + \varepsilon + \mu)E, \\ \dot{I}_1 &= \theta E + \omega_2 I_2 - (\omega_1 + k_1 + \mu + d_1)I_1, \\ \dot{I}_2 &= \varepsilon E + \omega_1 I_1 - (\omega_2 + k_2 + \mu + d_2)I_2, \\ \dot{R} &= k_1 I_1 + k_2 I_2 - \mu R.\end{aligned}\tag{2.1}$$

We assume that all the parameters are constants. Where δ is recruitment rate. β_1, β_2 are transmission rate for contact with the I_1 and I_2 class, respectively. μ is natural death rate, θ is rate of progression to I_1 class from the detected latent TB. ε is rate of progression to I_2 class from the detected latent TB. ω_1 is rate of progression to I_2 class from I_1 class. ω_2 is rate of progression to I_1 class from I_2 class. k_1 is rate of successfully treatment for active TB in the I_1 class. k_2 is rate of successfully

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