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Three solutions for a fractional Schrödinger equation with vanishing potentials*

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Abstract

In this paper, we study the following fractional Schrödinger equation

$$(-\Delta)^s u + V(x)u = K(x)f(u) + \lambda W(x)|u|^{p-2}u, \quad x \in \mathbb{R}^N,$$

where $\lambda > 0$ is a parameter, $(-\Delta)^s$ denotes the fractional Laplacian of order $s \in (0,1)$, $N > 2s$, $W \in L^{\frac{2}{2-p}}(\mathbb{R}^N, \mathbb{R}^+)$, $1 < p < 2$, V, K are nonnegative continuous functions and f is a continuous function with a quasicritical growth. Under some mild assumptions, we prove that the above equation has three solutions.

Keywords: Fractional Schrödinger equation, Vanishing potential, Variational methods.
2010 AMS Subject Classification: 35J20; 35A01; 58E05.

1 Introduction and the main results

This paper is concerned with the following fractional Schrödinger equation

$$(-\Delta)^s u + V(x)u = K(x)f(u) + \lambda W(x)|u|^{p-2}u, \quad x \in \mathbb{R}^N, \tag{1.1}$$

where $(-\Delta)^s$ denotes the fractional Laplacian of order $s \in (0,1)$, $N > 2s$, $\lambda > 0$, $V, K, f \in C(\mathbb{R}^N, \mathbb{R})$, $W \in L^{\frac{2}{2-p}}(\mathbb{R}^N, \mathbb{R}^+)$ and $1 < p < 2$.

In the last few years, the study of elliptic equation involving fractional Laplace operator appears widely in optimization, finance, phase transitions, stratified materials, crystal dislocation, flame propagation, conservation laws, materials science and water waves (see [4]). A basic motivation for the study of Eq. (1.1) arises in looking for the standing wave solutions of the type $\Psi(x,t) = e^{-iEt/\varepsilon}u(x)$ for the following time-dependent fractional Schrödinger equation

$$i\varepsilon \frac{\partial \Psi}{\partial t} = \varepsilon^{2s}(-\Delta)^s \Psi + (V(x) + E)\Psi - f(x, \Psi) \quad (x,t) \in \mathbb{R}^N \times \mathbb{R}. \tag{1.2}$$

Eq.(1.2) was introduced by Laskin [11, 12], which describes how the wave function of a physical system evolves over time. Over the past decades, problem (1.1) and problems similar as (1.1) have captured a lot of interest, many authors have shown their interest in elliptic equation and system both in bounded domains and unbounded domains, see [1, 5, 6, 10, 14].

Most of those results need to assume that the potential V admits a positive bounded from below. However, we point out that when $s = 1$, Ambrosetti, Felli and Malchiodi in [3] considered the zero mass case (i.e. $\lim_{|x|\to\infty} V(x) = 0$) for the problem

$$-\Delta u + V(x)u = K(x)|u|^p \ (1 < p < 2^* - 1),$$

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