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Analysis of the Gibbs phenomenon in stationary subdivision schemes

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Abstract

In this paper sufficient conditions to determine if a stationary subdivision scheme produces Gibbs oscillations close to discontinuities are presented. It consists on the positivity of the partial sums of the values of the mask. We apply the conditions to non-negative masks and analyze (numerically when the sufficient conditions are not satisfied) the Gibbs phenomenon in classical and recent subdivision schemes like B-splines, Deslauriers and Dubuc interpolation subdivision schemes and the schemes proposed in [S.S. Siddiqi and N. Ahmad. C^6 approximating subdivision scheme. Appl. Math. Letters 21, pp. 722-728 (2008)].

Keywords: Binary subdivision, Non-negative masks, B-spline subdivision schemes, Deslauriers-Dubuc subdivision schemes, 41A05, 41A10, 65D05, 65D17

1. Introduction and review

Subdivision schemes are powerful tools for generating curves and surfaces. They are used in many applications in different fields such as computer aided geometric design, computer animation or computer graphics. Following the notation used by Dyn and Levin in [4], a univariate stationary subdivision scheme with finitely support mask $\mathbf{a} = \{a_j\}_{j\in\mathbb{Z}}$ is defined as beginning with an initial sequence of finite data $f^0 = \{f_i^0\}_{i\in J_0}$. New values are obtained through refinement at level $k+1$, denoted by $f^{k+1} = \{f_i^{k+1}\}_{i\in J_{k+1}}$. It is the maximal set obtained by applying the rule:

$$(S_{\mathbf{a}} f^k)_i = f_i^{k+1} := \sum_{j\in J_k} a_{i-2j} f_j^k, \text{ then,} \quad J_{k+1} = 2J_k + supp(a) = \{i \in \mathbb{Z} : i = 2j + \gamma, j \in J_k, \gamma \in supp(a)\}.$$

There are two rules to define the points on the level $k+1$:

$$\begin{aligned} (S_{\mathbf{a}} f^k)_{2i} = f_{2i}^{k+1} &= \sum_{\gamma\in\mathbb{Z}} a_{2\gamma} f_{i-\gamma}^k, \qquad i \in \mathbb{Z}, \\ (S_{\mathbf{a}} f^k)_{2i+1} = f_{2i+1}^{k+1} &= \sum_{\gamma\in\mathbb{Z}} a_{2\gamma+1} f_{i-\gamma}^k, \qquad i \in \mathbb{Z}. \end{aligned} \tag{1}$$

We can represent these rules using algebraic formalism in terms of z-transforms. The symbol of the mask $\mathbf{a} = \{a_j\}_{j\in\mathbb{Z}}$ is defined as $a(z) = \sum_{j\in\mathbb{Z}} a_j z^j$. If $a^{[k]}$ is denoted as the k iterated symbol (see [4]), then, it is known that $a^{[k]}(z) = \prod_{l=1}^{k} a(z^{2^{l-1}})$. Therefore, if $0 \le l < 2^k$ then

$$f_{2^k i+l}^k = (S_{\mathbf{a}}^k f^0)_{2^k i+l} = (S_{\mathbf{a}^{[k]}} f^0)_{2^k i+l} = \sum_{\gamma\in\mathbb{Z}} a_{2^k\gamma+l}^{[k]} f_{i-\gamma}^0, \tag{2}$$

with

$$a_j^{[k]} = \sum_{i\in\mathbb{Z}} a_i^{[k-1]} a_{j-2i}, \tag{3}$$

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