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A CRITICAL POINT APPROACH TO BOUNDARY-VALUE PROBLEMS ON THE REAL LINE

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ABSTRACT. We discuss the existence of at least one weak solution for elliptic problems on the real line. Our technical approach is based on variational methods. Some recent results are extended and improved. Examples are presented to demonstrate the application of our main results.

1. INTRODUCTION

Boundary value problems on infinite intervals frequently occur in mathematical modelling of various applied problems, for example, in the study of unsteady flow of a gas through a semi-infinite porous medium [13], discussion of electrostatic probe measurements in solid-propellant rocket exhausts [15], analysis of the mass transfer on a rotating disk in a non-Newtonian fluid [16], heat transfer in the radial flow between parallel circular disks [16], as well as numerous problems arising in the study of draining flows [3], plasma physics [2] and radially symmetric solutions of semilinear elliptic equations [2]. Recently, many researchers have paid more attention to the boundary-value problems on unbounded intervals, for instance see [5, 7–9, 11, 14] and the references therein, and it is worth to mention that among them only in [5, 8, 9] boundary-value problems on the whole line were studied.

In this paper, we consider the elliptic problem on the real line

$$-(|u'(x)|^{p-2}u'(x))' + B|u(x)|^{p-2}u(x) = \lambda\alpha(x)g(u(x)) \quad \text{for a.e. } x \in \mathbb{R}, \tag{1.1}$$

where λ is a real positive parameter, B is a real positive number, and $\alpha, g : \mathbb{R} \to \mathbb{R}$ are two functions such that $\alpha \in L^1(\mathbb{R})$, $\alpha(x) \geq 0$ for a.e. $x \in \mathbb{R}, \alpha \not\equiv 0$ on any subset of positive measure in $\mathbb{R}$ and g is continuous. In fact, we exploit a smooth version of [6, Theorem 2.1] which is a more precise version of Ricceri's variational principle [17] in order to ensure the existence of a precise open interval of positive eigenvalues for which the treated problem admits at least one nontrivial weak solution and for sufficiently small λ, the energy functional I_λ related to the problem is negative and decreasing on the solutions; see Theorem 3.1.

A special case of our main result, Theorem 3.1, is the following theorem.

Theorem 1.1. *Assume that* $g(0) = 0$, $0 < \sup_{\theta>0} \frac{\theta^2}{\sup_{|\xi|\leq\theta}\int_0^\xi g(t)\mathrm{d}t} < \infty$ *and* $\lim_{\xi\to 0^+} \frac{\int_0^\xi g(t)\mathrm{d}t}{|\xi|^2} = \infty$. *Then, for each* $\lambda \in \overline{\Lambda} := \left(0, \frac{1}{\int_{\mathbb{R}}\alpha(x)\mathrm{d}x} \sup_{\theta>0} \frac{\theta^2}{\sup_{|\xi|\leq\theta}\int_0^\xi g(t)\mathrm{d}t}\right)$,

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