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Existence and Uniqueness of Normalized Solutions for the Kirchhoff equation

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Abstract

For a class of Kirchhoff functional, we first give a complete classification with respect to the exponent p for its L^2-normalized critical points, and show that the minimizer of the functional, if exists, is unique up to translations. Secondly, we search for the mountain pass type critical points for the functional on the L^2-normalized manifold, and also prove that this type critical point is unique up to translations. Our proof relies only on some simple energy estimates and avoids using the concentration-compactness principles. These conclusions extend some known results in previous papers.

Keywords: L^2-normalized critical point; Kirchhoff equation; Uniqueness
MSC: 35J20; 35J60

1 Introduction

In this paper, we study the following Kirchhoff equation

$$-\Big(a+b\int_{\mathbb{R}^N}|\nabla u|^2dx\Big)\Delta u-|u|^pu=\mu u \text{ in } \mathbb{R}^N, \tag{1.1}$$

where $a,b>0, 1\le N\le 3$, and $0<p<2^*-2$ with $2^*=+\infty$ if $N=1,2$, or $2^*=6$ if $N=3$. Equation (1.1) is related to the stationary solutions of

$$u_{tt}-\Big(a+b\int_{\mathbb{R}^N}|\nabla u|^2dx\Big)\Delta u=f(x,u), \tag{1.2}$$

where $f(x,u)$ is a general nonlinearity. The problem (1.2) was proposed by Kirchhoff [8] and models free vibrations of elastic strings by taking into account the changes in length of the string produced by transverse vibrations. Comparing with the semilinear equations (i.e., setting $b=0$ in above two equations), it is much more challenging and interesting to investigate equations (1.1) and (1.2) in view of the existence of the nonlocal term $\big(\int_{\mathbb{R}^N}|\nabla u|^2dx\big)\Delta u$.

After the pioneering work of [11] and [10], much attention was paid to these above two equations. For instance, replacing the term $|u|^pu$ with a general nonlinearity $f(x,u)$, there are many results on the existence of solutions for equation (1.1), one can refer [2, 4, 5, 9] and the references therein. Equation (1.1) can be viewed as an eigenvalue problem by taking μ as an unknown Lagrange multiplier. From this point of view, one can solve (1.1) by studying some constrained variational problems and obtain normalized solutions. Motivated by the works of [1, 13], we consider the following minimization problem:

$$I(c):=\inf_{u\in S_c}E(u), \tag{1.3}$$

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