Accepted Manuscript

A note on a family of non-gravitational central force potentials in dimension one

M. Alvarez-Ramírez, M. Corbera, Josep M. Cors, A. García

PII: S0893-9659(17)30133-7
DOI: http://dx.doi.org/10.1016/j.aml.2017.04.020
Reference: AML 5241

To appear in: *Applied Mathematics Letters*

Received date : 12 March 2017
Accepted date : 19 April 2017

Please cite this article as: M. Alvarez-Ramírez, et al., A note on a family of non-gravitational central force potentials in dimension one, Appl. Math. Lett. (2017), http://dx.doi.org/10.1016/j.aml.2017.04.020

A note on a family of non-gravitational central force potentials in dimension one

M. Alvarez-Ramírez[a], M. Corbera[b], Josep M. Cors[c], A. García[a,*]

[a] *Departamento de Matemáticas, UAM–Iztapalapa, 09340 Iztapalapa, México City, México*
[b] *Facultat de Ciències i Tecnologia. Universitat de Vic-Universitat Central de Catalunya (UVic-UCC), C. de la Laura, 13, 08500 Vic, Spain*
[c] *Departament de Matemàtiques , Universitat Politècnica de Catalunya, 08242 Manresa (Barcelona), Spain.*

Abstract

In this work we study a one-parameter family of differential equations and the different scenarios that arise with the change of parameter. We remark that these are not bifurcations in the usual sense but a wider phenomenon related with changes of continuity or differentiability. We offer an alternative point of view for the study for the motion of a system of two particles which will always move in some fixed line, we take $\mathbb{R}$ for the position space. If we fix the center of mass at the origin, so the system reduces to that of a single particle of unit mass in a central force field. We take the potential energy function $U(x) = |x|^\beta$, where x is the position of the single particle and β some positive real number.

Keywords: Singularities; collisions; non-gravitational interactions.

1. Introduction

In 1981, R. McGehee [1] investigated geometrically the regularization of binary collisions of classical particle systems with non-gravitational interactions. R. McGehee considered the motion near a collision of a particle in the vector field given by the homogeneous potential $U(x) = -|x|^{-\alpha}$, where $x \in \mathbb{R}^2$ is the position of a single particle and α is a positive real number. McGehee showed by appropriate coordinate transformations that the singularity corresponding to a double collision ($x = 0$) is blown up to a collision manifold, after that the time variable is rescaled appropriately, and finally the vector field is extended smoothly to this manifold. He noted that there exits a bifurcation at $\alpha = 2$.

More recently, Xia and Jardón-Kojakhmetov [2] investigated the topological structure of the same system as α varies along the entire real line $\mathbb{R}$. This study

*Corresponding author
Email addresses: mar@xanum.uam.mx (M. Alvarez-Ramírez), montserrat.corbera@uvic.cat (M. Corbera), cors@epsem.upc.edu (Josep M. Cors), agar@xanum.uam.mx (A. García)