Accepted Manuscript

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PII: S0893-9659(17)30205-7
DOI: http://dx.doi.org/10.1016/j.aml.2017.06.003
Reference: AML 5278

To appear in: *Applied Mathematics Letters*

Received date : 15 April 2017
Revised date : 5 June 2017
Accepted date : 5 June 2017

Please cite this article as: H. Jin, W. Liu, Fractional Kirchhoff equation with a general critical nonlinearity, Appl. Math. Lett. (2017), http://dx.doi.org/10.1016/j.aml.2017.06.003

Fractional Kirchhoff equation with a general critical nonlinearity

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Abstract

In this paper, *for any dimension* $N > 2s(0 < s < 1)$, we study the fractional Kirchhoff equation

$$\left(a + b\int_{\mathbb{R}^N} |(-\Delta)^{\frac{s}{2}}u|^2 dx\right)(-\Delta)^s u + u = f(u) \ \text{ in } \ \mathbb{R}^N,$$

with a critical nonlinearity, where $(-\Delta)^s$ is the fractional Laplacian. By using a perturbation approach, we prove the existence of solutions to the above problem without the Ambrosetti-Rabinowitz condition when the parameter b is small. What's more, we obtain the asymptotic behavior of solutions as $b \to 0$. The method we use and the result we get are applicable *in any dimension* $N > 2s$. Our result improves the study made in the low dimension $2s < N < 4s$.

Keywords: fractional Kirchhoff equation, variational methods, critical growth
2010 MSC: 35A15, 35B33, 35J60

1. Introduction and main result

In this paper, we are concerned with the following fractional Kirchhoff equation

$$\left(a + b\int_{\mathbb{R}^N} |(-\Delta)^{\frac{s}{2}}u|^2 dx\right)(-\Delta)^s u + u = f(u) \ \text{ in } \ \mathbb{R}^N, \tag{1.1}$$

where $N > 2s$ with $0 < s < 1$, a, b are positive constants and $(-\Delta)^s u$ is the fractional Laplacian which arises in the description of various phenomena in the applied science, such as the phase transition [19], Markov processes [1] and fractional quantum mechanics [15]. When $a = 1$ and $b = 0$, (1.1) becomes the fractional Schrödinger equations which have been studied by many authors.

We refer the readers to [2, 5, 6, 7] and the references therein for the details. When $s = 1$, the problem (1.1) reduces to the well-known Kirchhoff equation

$$-\left(a + b\int_{\mathbb{R}^N} |\nabla u|^2 dx\right)\Delta u + u = f(u) \ \text{ in } \ \mathbb{R}^N, \tag{1.2}$$

which has been studied in the last decade, see [9, 12, 17]. The equation (1.2) is related to the stationary analogue of the Kirchhoff equation $u_{tt} - \left(a + b\int_\Omega |\nabla u|^2 dx\right)\Delta u = f(x, u)$ on $\Omega \subset \mathbb{R}^N$ bounded, which was proposed by Kirchhoff [13] in 1883 as a generalization the classic D'Alembert's wave equation for free vibrations of elastic strings. Recently, in bounded regular domains of $\mathbb{R}^N$, Fiscella and Valdinoci [11] proposed the following fractional stationary Kirchhoff equation

$$\begin{cases} M\left(\int_{\mathbb{R}^N} |(-\Delta)^{\frac{s}{2}}u|^2\right)(-\Delta)^s u = f(x, u), & \text{in } \ \Omega, \\ u = 0 \quad \text{in } \ \mathbb{R}^N \backslash \Omega, \end{cases} \tag{1.3}$$

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