

## Accepted Manuscript

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PII: S0893-9659(17)30205-7  
DOI: <http://dx.doi.org/10.1016/j.aml.2017.06.003>  
Reference: AML 5278

To appear in: *Applied Mathematics Letters*

Received date: 15 April 2017  
Revised date: 5 June 2017  
Accepted date: 5 June 2017

Please cite this article as: H. Jin, W. Liu, Fractional Kirchhoff equation with a general critical nonlinearity, *Appl. Math. Lett.* (2017), <http://dx.doi.org/10.1016/j.aml.2017.06.003>

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# Fractional Kirchhoff equation with a general critical nonlinearity

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## Abstract

In this paper, for any dimension  $N > 2s$  ( $0 < s < 1$ ), we study the fractional Kirchhoff equation

$$\left( a + b \int_{\mathbb{R}^N} |(-\Delta)^{\frac{s}{2}} u|^2 dx \right) (-\Delta)^s u + u = f(u) \text{ in } \mathbb{R}^N,$$

with a critical nonlinearity, where  $(-\Delta)^s$  is the fractional Laplacian. By using a perturbation approach, we prove the existence of solutions to the above problem without the Ambrosetti-Rabinowitz condition when the parameter  $b$  is small. What's more, we obtain the asymptotic behavior of solutions as  $b \rightarrow 0$ . The method we use and the result we get are applicable in any dimension  $N > 2s$ . Our result improves the study made in the low dimension  $2s < N < 4s$ .

*Keywords:* fractional Kirchhoff equation, variational methods, critical growth

*2010 MSC:* 35A15, 35B33, 35J60

## 1. Introduction and main result

In this paper, we are concerned with the following fractional Kirchhoff equation

$$\left( a + b \int_{\mathbb{R}^N} |(-\Delta)^{\frac{s}{2}} u|^2 dx \right) (-\Delta)^s u + u = f(u) \text{ in } \mathbb{R}^N, \quad (1.1)$$

where  $N > 2s$  with  $0 < s < 1$ ,  $a, b$  are positive constants and  $(-\Delta)^s u$  is the fractional Laplacian which arises in the description of various phenomena in the applied science, such as the phase transition [19], Markov processes [1] and fractional quantum mechanics [15]. When  $a = 1$  and  $b = 0$ , (1.1) becomes the fractional Schrödinger equations which have been studied by many authors.

We refer the readers to [2, 5, 6, 7] and the references therein for the details. When  $s = 1$ , the problem (1.1) reduces to the well-known Kirchhoff equation

$$-\left( a + b \int_{\mathbb{R}^N} |\nabla u|^2 dx \right) \Delta u + u = f(u) \text{ in } \mathbb{R}^N, \quad (1.2)$$

which has been studied in the last decade, see [9, 12, 17]. The equation (1.2) is related to the stationary analogue of the Kirchhoff equation  $u_{tt} - (a + b \int_{\Omega} |\nabla u|^2 dx) \Delta u = f(x, u)$  on  $\Omega \subset \mathbb{R}^N$  bounded, which was proposed by Kirchhoff [13] in 1883 as a generalization the classic D'Alembert's wave equation for free vibrations of elastic strings. Recently, in bounded regular domains of  $\mathbb{R}^N$ , Fiscella and Valdinoci [11] proposed the following fractional stationary Kirchhoff equation

$$\begin{cases} M \left( \int_{\mathbb{R}^N} |(-\Delta)^{\frac{s}{2}} u|^2 \right) (-\Delta)^s u = f(x, u), & \text{in } \Omega, \\ u = 0 & \text{in } \mathbb{R}^N \setminus \Omega, \end{cases} \quad (1.3)$$

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