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Error estimates of spectral element methods with generalized Jacobi polynomials on an interval[☆]

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Abstract

Based on the generalized Jacobi polynomials with indices $(-1,-1)$, a new upper bound for a posteriori error estimates is proposed, investigated and implemented for generalized Jacobi-Galerkin spectral element approximations. To simplify discussion for the error estimates, the second-order partial differential equation with homogeneous Dirichlet boundary conditions is considered on a unit interval.

Keywords: Generalized Jacobi polynomial, weighted orthogonality, spectral element method, a posteriori error estimate

1. Introduction

Over the past decades, spectral element methods have been playing increasingly a significant role in engineering computations and scientific simulations. The most important advantage of the spectral element method is that, by some orthogonal properties of given polynomials, the stiff or mass matrix is exactly diagonal. Hence one can use explicit integration schemes to drastically simplify the implementation and reduce the computational cost. The key point is that this method avoids inverting the discretized approximation system.

Since Patera [1] proposed the spectral element method, combining the flexibility of the finite element method with the accuracy of spectral techniques, to solve the incompressible Navier-Stokes equations, there have been active research on spectral element methods (see, for instance, [2, 3, 4, 5, 6, 7, 8, 9] and the references cited therein). Zayernouri and Karniadakis [10] employed spectral element methods to get exponentially accurate numerical solutions for fractional ordinary differential equations. In the literatures, most of the researches and studies focused on the classical Jacobi polynomials, denoted by $J_k^{\alpha_1,\alpha_2}$ $(k \geq 0, \alpha_1, \alpha_2 > -1)$. Shen [11] introduced an efficient dual-Petrov-Galerkin method for third and higher odd-order differential equations by Jacobi polynomials with weighted indices $(0, 0)$, *i.e.* the Legendre polynomials. The authors based on the Jacobi polynomials to study the spectral methods for Volterra integral equations, such as [12, 13] and their references. It is very useful to study numerical methods based on Jacobi polynomials with general negative indices, *i.e.*, the generalized Jacobi polynomials, which are with indices $\alpha_1, \alpha_2 \in \mathbb{R}$. In fact, the generalized Jacobi polynomials were introduced in [14, 15, 16] with details. The generalized Jacobi polynomials with negative indices not only can be directly used to simplify the numerical analysis for the spectral and spectral element approximations of differential equations, but also lead to very efficient numerical algorithm (see, for instance, [14, 15] and the references therein).

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