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A note on the stationary distribution of the stochastic chemostat model with general response functions

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Abstract A model of the chemostat involving stochastic perturbation is considered. Instead of assuming the familiar Monod kinetics for nutrient uptake, a general class of functions is used which includes both monotone and non-monotone uptake functions. Using the stochastic Lyapunov analysis method, under restrictions on the intensity of the noise, we show the existence of a stationary distribution and the ergodicity of the stochastic system.

Keywords Chemostat; General response functions; Ergodicity; Stationary distribution.

1 Introduction

The chemostat, a basic laboratory apparatus used for the continuous culture of microorganisms, has played an important role in many fields. Thus it occupies a central place both in mathematical and theoretical ecology (see Weedermann et al. [1], Zhang et al. [2], Meng et al. [3] and the references therein for the newest research on chemostat). The classic chemostat with single species and single substrate can be expressed in the form

$$\begin{cases} \dfrac{dS(t)}{dt} = D(S^0 - S(t)) - \dfrac{1}{\delta}p(S(t))x(t), \\ \dfrac{dx(t)}{dt} = -Dx(t) + p(S(t))x(t), \end{cases} \tag{1.1}$$

where $S(t)$, $x(t)$ are the concentrations of the nutrient and the microbial population at time t, respectively; S^0 is the original input of nutrient and D is the common dilution rate. The term $\frac{1}{\delta}p(S)$ denotes the uptake rate of the substrate by the microbial population. $p(S)$ represents the per-capita growth rate of the species and δ is a growth yield constant. The growth response function $p: \mathbb{R}_+ \to \mathbb{R}_+$ is generally assumed to satisfy

$$p \text{ is continuously differentiable,} \tag{1.2}$$

$$p(0) = 0, \ p(S) > 0 \text{ for } S > 0. \tag{1.3}$$

Butler et al. [4] investigated the global dynamics of the chemostat with a general class of functions describing nutrient uptake. In the single-species case, there exists a uniquely defined positive real number $0 < \lambda \leq \infty$ such that $p(S) < D$ for $0 < S < \lambda$; $p(S) > D$ for $S > \lambda$. Here λ represents the break-even concentration of the substrate for the species $x(t)$. If $\lambda < S^0$, the solution of system (1.1) satisfies

$$\lim_{t\to\infty} S(t) = \lambda, \quad \lim_{t\to\infty} x(t) := x^* = \delta(S^0 - \lambda).$$

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