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On asymptotic relationships between two higher order dynamic equations on time scales

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ABSTRACT

We consider the nth order dynamic equations $x^{\Delta^n}+p_1(t)x^{\Delta^{n-1}}+\cdots+p_n(t)x=0$ and $y^{\Delta^n}+p_1(t)y^{\Delta^{n-1}}+\cdots+p_n(t)y=f(t,y(\tau(t)))$ on a time scale $\mathbb{T}$, where τ is a composition of the forward jump operators, p_i are real rd-continuous functions and f is a continuous function; $\mathbb{T}$ is assumed to be unbounded above. We establish conditions that guarantee asymptotic equivalence between some solutions of these equations. No restriction is placed on whether the solutions are oscillatory or nonoscillatory. Applications to second order Emden–Fowler type dynamic equations and Euler type dynamic equations are shown.

1. Introduction

Let us consider the delta-differential operator $\mathcal{L}$ defined by $\mathcal{L}[x](t)=x^{\Delta^n}+p_1(t)x^{\Delta^{n-1}}+\cdots+p_n(t)x$, where $n\in\mathbb{N}$ and p_i, $i=1,\ldots,n$, are real rd-continuous functions on the time scale interval $[t_0,\infty)_{\mathbb{T}}\subseteq\mathbb{T}$. The time scale $\mathbb{T}$ is assumed to be unbounded from above. We study asymptotic relationships between the solutions of the nth order linear dynamic equation

$$\mathcal{L}[x](t)=0 \tag{1}$$

and the nonlinear (in general) dynamic equation

$$\mathcal{L}[y](t)=f(t,y(\tau(t))), \tag{2}$$

where $f:[t_0,\infty)_{\mathbb{T}}\times\mathbb{R}\to\mathbb{R}$ is a continuous function and $\tau=\sigma\circ\cdots\circ\sigma$ (m-times), $m\in\mathbb{N}_0$, with $\tau=\mathrm{id}$ when $m=0$. We establish conditions that guarantee an asymptotic equivalence between certain of the solutions of (1) and certain of the solutions of (2). We emphasize that no restriction is placed on whether solutions of (1) are nonoscillatory (i.e., eventually of one sign) or oscillatory (i.e., not nonoscillatory).

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We use the standard notation concerning time scales calculus: f^{Δ}, f^{Δ^n}, σ, μ, $\int_a^b f(s)\,\Delta s$, $[a,b)_{\mathbb{T}}$, and $e_u(t,s)$ stand for delta-derivative of f, nth delta-derivative of f, the forward jump operator, graininess, delta integral, time scale interval, and generalized exponential function, respectively; for these definitions, see e.g. [1].

2. Main results

By a solution u of (1) or (2) we mean an n-times delta-differentiable function that satisfies $\mathcal{L}[u](t) = 0$ or $\mathcal{L}[u](t) = f(t, u(\tau(t)))$ for all $t \in [\bar{t}, \infty)_{\mathbb{T}}$ with some $\bar{t} \in [t_0, \infty)_{\mathbb{T}}$. In addition to the rd-continuity of the coefficients p_i, we assume the condition

$$1 + \sum_{i=1}^{n} (-\mu(t))^i p_i(t) \neq 0 \quad \text{for all } t \in [t_0, \infty)_{\mathbb{T}}.$$

Then linear equation (1) is regressive, and the initial value problem for (1) is uniquely globally solvable, see [1, Corollary 5.90, Theorem 5.91].

We define the time scale Wronskian W of the set $\{x_1, \dots, x_j\}$ of $(j-1)$-times delta differentiable functions by

$$W(x_1, \dots, x_j) = \begin{vmatrix} x_1 & x_2 & \cdots & x_j \\ x_1^{\Delta} & x_2^{\Delta} & \cdots & x_j^{\Delta} \\ \vdots & \vdots & & \vdots \\ x_1^{\Delta^{j-1}} & x_2^{\Delta^{j-1}} & \cdots & x_j^{\Delta^{j-1}} \end{vmatrix}.$$

The function $W(t) = W(x_1, \dots, x_n)(t)$ satisfies the equation

$$W^{\Delta} = -\left\{ \sum_{i=1}^{n} (-\mu(t))^{i-1} p_i(t) \right\} W \tag{3}$$

provided $\{x_1, \dots, x_n\}$ is a fundamental set of (1). For further properties of the Wronskian associated to (1) see [1, Section 5.5].

The following hypotheses will be utilized in the sequel. Let $\{x_1, \dots, x_n\}$ be a (fixed) fundamental system of (1). Denote $\omega_i = W(x_1, \dots, x_{i-1}, x_{i+1}, \dots, x_n)/W(x_1, \dots, x_n)$. Assume that there exist positive rd-continuous functions $\widehat{x}_i, h_i$, $1 \le i \le n$, such that

$$|x_i(t)| \le \widehat{x}_i(t), \quad t \in [t_0, \infty)_{\mathbb{T}},\ 1 \le i \le n, \tag{4}$$

and

$$|\omega_i(t)| \le h_i(t), \quad t \in [t_0, \infty)_{\mathbb{T}},\ 1 \le i \le n. \tag{5}$$

Let f satisfy the inequality

$$|f(t, y)| \le G(t, |y|) \quad \text{on } [t_0, \infty)_{\mathbb{T}} \times \mathbb{R}, \tag{6}$$

where $G : [t_0, \infty)_{\mathbb{T}} \times [0, \infty) \to [0, \infty)$ is continuous and $u \mapsto G(t, u)$ is nondecreasing.

For $g_1, g_2 : \mathbb{T} \to (0, \infty)$ we write $g_1(t) = o(g_2(t))$ as $t \to \infty$ and $g_1(t) = O(g_2(t))$ as $t \to \infty$ when $\lim_{t\to\infty} g_1(t)/g_2(t) = 0$ and there exists $M \in (0, \infty)$ such that $g_1(t) \le M g_2(t)$ for large t, respectively.