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A two-mode modified KdV equation with multiple soliton solutions

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ABSTRACT

In this work we establish a two-mode modified Korteweg–de Vries equation (TmKdV). We show that multiple soliton solutions exist for specific values of the nonlinearity and dispersion parameters of this equation. We also derive more exact solutions for other values of these parameters. We will use the simplified Hirota's method, the tanh/coth method to conduct this analysis.

1. Introduction

The celebrated KdV equation

$$u_t + auu_x + u_{xxx} = 0, \tag{1}$$

and the modified KdV (mKdV) equation

$$u_t + au^2u_x + u_{xxx} = 0, \tag{2}$$

are the pioneer equations that highly contributed for the development of the solitary waves theory. The KdV equation is a classical paradigm of integrable nonlinear evolution equations that arises in many physical phenomena such as ion-acoustic waves in plasmas and surface water waves. The mKdV equation arises in electric circuits and multi-component plasmas. Many aspects of nonlinear wave theory have been developed over the past decades.

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In [1], Korsunsky was the first to propose the nonlinear dispersive two-mode KdV equation, referred to as the two-mode KdV equation (TKdV), defined as

$$\begin{aligned} &u_{tt} + (c_1 + c_2)u_{xt} + c_1 c_2 u_{xx} + \left((\alpha_1 + \alpha_2)\frac{\partial}{\partial t} + (\alpha_1 c_2 + \alpha_2 c_1)\frac{\partial}{\partial x}\right) u u_x \\ &\quad + \left((\beta_1 + \beta_2)\frac{\partial}{\partial t} + (\beta_1 c_2 + \beta_2 c_1)\frac{\partial}{\partial x}\right) u_{xxx} = 0, \end{aligned} \tag{3}$$

where $u(x,t)$ is a field function, $-\infty < x, t < \infty$, the α_i are the parameters of nonlinearity, and the β_i are the dispersion parameters for the first ($i = 1$) and for the second ($i = 2$) mode [1], and the c_i are the phase velocities. The field function $u(x,t)$ represents the height of the water's free surface above a flat bottom. The TKdV equation describes the propagation of two different wave modes in the same direction simultaneously, with the same dispersion relation but different phase velocities, nonlinearity, and dispersion parameters [1–7].

In [1–7], by using specific transformations, Eq. (3) was reduced to

$$u_{tt} - s^2 u_{xx} + \left(\frac{\partial}{\partial t} - \alpha s \frac{\partial}{\partial x}\right) u u_x + \left(\frac{\partial}{\partial t} - \beta s \frac{\partial}{\partial x}\right) u_{xxx} = 0, \tag{4}$$

where

$$\begin{aligned} s &= \frac{1}{2}(c_1 - c_2), \\ \alpha &= \frac{\alpha_2 - \alpha_1}{\alpha_2 + \alpha_1}, |\alpha| \le 1, \\ \beta &= \frac{\beta_2 - \beta_1}{\beta_2 + \beta_1}, |\beta| \le 1. \end{aligned} \tag{5}$$

It is to be noted that for $s = 0$, the TKdV equation (4) is reduced to the standard KdV equation after integrating with respect to time t.

Various methods have been used to investigate the nonlinear evolutions [1–12], such as the Hirota bilinear method and its simplified version, and the Bäcklund transformation method. The Hirota's bilinear method and the simplified form possess strong features that make it practical for the determination of multiple soliton solutions.

The main goals set for this work are two-fold. We first aim to establish a two-mode modified KdV equation (TmKdV). Second, we seek to conduct a reliable analysis to examine the conditions that will make this newly developed equation give multiple soliton solutions.

2. Formulation of the two-mode modified KdV equation

To establish the two-mode modified KdV equation, we combine the sense of Korsunsky [1] used to propose the TKdV equation (3), and the structure of the modified KdV equation (2), to propose the nonlinear dispersive equation, and will be referred to it as the two-mode mKdV equation (TmKdV), given as

$$\begin{aligned} &u_{tt} + (c_1 + c_2)u_{xt} + c_1 c_2 u_{xx} + \left((\alpha_1 + \alpha_2)\frac{\partial}{\partial t} + (\alpha_1 c_2 + \alpha_2 c_1)\frac{\partial}{\partial x}\right) u^2 u_x \\ &\quad + \left((\beta_1 + \beta_2)\tfrac{\partial}{\partial t} + (\beta_1 c_2 + \beta_2 c_1)\tfrac{\partial}{\partial x}\right) u_{xxx} = 0, \end{aligned} \tag{6}$$

where $u(x,t)$ is a field function, $-\infty < x, t < \infty$. We define the parameters α_i as the nonlinearity parameters, and the parameters β_i as the dispersion parameters for the first ($i = 1$) and for the second ($i = 2$) mode [1], and the c_i are the phase velocities. The field function $u(x,t)$ represents the height of the water's free surface above a flat bottom. The TmKdV equation describes the propagation of two different wave modes in the same direction simultaneously.