Applied Mathematics Letters 70 (2017) 7–13

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

Asymptotic behaviour of a population model with piecewise constant argument

Fatma Karakoç

Department of Mathematics, Faculty of Sciences, Ankara University, 06100 Ankara, Turkey

ARTICLE INFO

Article history:
Received 6 January 2017
Received in revised form 17 February 2017
Accepted 18 February 2017
Available online 24 February 2017

Keywords:
Piecewise constant argument
Difference equation
Linearized oscillation
Nonoscillation

ABSTRACT

We investigate oscillation about the positive equilibrium point of a population model with piecewise constant argument. By using linearized oscillation theory for difference equations a necessary and sufficient condition for the oscillation is obtained. Moreover it is showed that every nonoscillatory solution approaches to the equilibrium point as t tends to infinity.

1. Introduction

This paper is concerned with the global asymptotic behaviour of the positive solutions of a functional differential equation of the form

$$N'(t) = -\gamma N(t) + \frac{\beta N(t)}{r + N^m(\lfloor t-1 \rfloor)}, \tag{1}$$

where $\gamma, \beta, r \in (0, \infty)$, $m \in \mathbb{N} = \{1, 2, \ldots\}$ and $\lfloor . \rfloor$ denotes the floor function which gives the greatest integer less than or equal to given number. The mathematical model

$$x'(t) + px(t) - q\frac{x(t)}{r + x^n(t-\tau)} = 0 \tag{2}$$

which is related to control of a single population of cells was proposed by Nazarenko [1]. Oscillation and stability of Eq. (2) was studied in [2].

On the other hand, since 1980s differential equations with piecewise constant arguments have attracted great deal of attention of researchers in mathematical and some of the others fields in science. In 1984,

E-mail address: fkarakoc@ankara.edu.tr.

http://dx.doi.org/10.1016/j.aml.2017.02.014
0893-9659/

Cooke and Wiener [3] studied oscillatory and periodic solutions of a linear differential equation with piecewise constant argument and they note that such equations are comprehensively related to impulsive and difference equations. After this work, oscillatory and periodic solutions of linear differential equations with piecewise constant arguments have been dealt with by many authors [4–10] and the references cited therein. In 1990, Gopalsamy et. all [11] studied the oscillatory properties of the logistic equation with piecewise constant arguments

$$\frac{dN}{dt} = rN(t)\left\{1 - \sum_{j=0}^{m} p_j N([t-j])\right\}.$$

In 2013, sufficient conditions for the oscillation of a nonlinear impulsive differential equation with piecewise constant argument were obtained [12]. So, as we know, there are only a few works on the oscillation of nonlinear differential equations with piecewise constant arguments which are important for applications. This situation is the main motivation for us to investigate Eq. (1). This paper is organized as follows. In Section 2, we give some basic definitions as well as useful well known results. In Section 3, we obtain main results and we give some examples to illustrate our results.

2. Preliminaries

In this section we present some definitions and results.

Definition 1. It is said that a function N defined on the set $\{-1\} \cup [0, \infty)$ is a solution of Eq. (1) if it satisfies the following conditions:

(D_1) $N(t)$ is continuous on $\mathbb{R}^+$,

(D_2) $N(t)$ is differentiable and satisfies (1) for any $t \in \mathbb{R}^+$, with the possible exception of the points $\lfloor t \rfloor$ in $\mathbb{R}^+$ where one-sided derivatives exist.

By the biological meaning of the model (1), we only consider positive solutions. So, we consider Eq. (1) with the initial conditions

$$N(-1) = N_{-1} > 0,\ N(0) = N_0 > 0. \tag{3}$$

Throughout this paper we assume that $\frac{\beta}{\gamma} > r$. Then, by the method of steps it can be easily shown that all solutions of the initial value problem (1), (3) are positive.

Moreover, it is clear that $N^* = \left(\frac{\beta}{\gamma} - r\right)^{1/m}$ is the positive equilibrium point of Eq. (1). By setting $N(t) = N^* e^{x(t)}$, the following equation is obtained:

$$x'(t) = -\gamma + \frac{\beta}{r + (N^*)^m e^{mx(\lfloor t-1 \rfloor)}}. \tag{4}$$

Definition 2. A function $x(t)$ is called oscillatory about N^* if the function $(x(t) - N^*)$ has arbitrarily large zeros. A function $x(t)$ is said oscillatory if it oscillates about zero. Otherwise, the function $x(t)$ is called nonoscillatory.

Remark 1. It is clear that the solution $N(t)$ of Eq. (1) oscillates about N^* if and only if the solution $x(t)$ of Eq. (4) oscillates about zero.

Definition 3. The sequence $\{y_n\}$ is said oscillatory if it is neither eventually positive nor eventually negative. Otherwise, it is called nonoscillatory.