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On (non-)exponential decay in generalized thermoelasticity with two temperatures

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Abstract

We study solutions for the one-dimensional problem of the Green-Lindsay and the Lord-Shulman theories with two temperatures. First, existence and uniqueness of weakly regular solutions are obtained. Second, we prove the exponential stability in the Green-Lindsay model, but the non-exponential stability for the Lord-Shulman model.

Keywords: Two-temperatures; generalized thermoelasticity; exponential decay; Green-Lindsay theory; Lord-Shulman theory

1 Introduction

The usual theory of heat conduction based on Fourier's law implies the instantaneous propagation of heat waves. This fact is not well accepted from the viewpoint of physics because it contradicts the causality principle. Accordingly, a big interest has been developed to propose alternative constitutive equations to the Fourier law. We recall the classical formulations of Lord-Shulman [9] and Green-Lindsay theories [5], which are based on the Cattaneo-Maxwell theory heat conduction. This is the case when the heat equation is hyperbolic.

Thermoelasticity with two temperatures is one of the non-classical theories of thermomechanics of elastic solids. The main difference of this theory with respect to the classical one is in the thermal dependence. The theory was proposed by Chen, Gurtin and Williams (see [1], [3], [6]) and several authors have dedicate its attention to this problem (Ieşan [7], Chen et al. [2], [16], Quintanilla [11], [12], among others). In this paper where elastic effects are taken into consideration we deal with the two models proposed by Youssef [18]. They correspond to the two-temperature modifications of the Green-Lindsay and Lord-Shulman theories. Uniqueness and instability of solutions was obtained in [10].

First, the well-posedness will be proved in spaces with only combined, hence less regularity than known for the classical single-temperature case. Then we prove that the solutions uniformly decay exponentially for the Green-Lindsay theory, but the decay is slow – not exponential - for the Lord-Shulman case. This is a surprising aspect of this paper providing another interesting example for a situation where the change from Fourier's to Cattaneo-Maxwell's law leads to a loss of exponential stability, cp. [14] for the classical exponentially stable single-temperature case, and [4, 13, 15] for other examples of loss of exponential stability for plates or Timoshenko type models.