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Almost periodic solution for a new type of neutral impulsive stochastic Lasota-Wazewska timescale model*

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Abstract

In this paper, we propose two new concepts of mean-square almost periodic stochastic process based on a new concept of periodic time scales introduced by M. Adıvar (Math. Slovaca, 63 (2013), 817-828). Then we provide some sufficient conditions to guarantee the existence of mean-square almost periodic solution for a new type of neutral impulsive stochastic Lasota-Wazewska model involving q-difference model on time scales.

Keywords: Time scales, Mean-square almost periodic solution, Piecewise stochastic process, Lasota-Wazewska model, q-difference.
2010MSC: 34N05, 34C27, 34K45, 60H10, 91B70.

1 Introduction and model description

In 1999, Gopalsamy and Trofimchuk [1] studied the existence of an almost periodic solution of the Lasota-Wazewska type delay differential equation which was used by Wazewska-Czyzewska and Lasota [2] as a model for the survival of red blood cells in an animal. Since then periodic and almost periodic solutions for this classical model have been widely generalized to discrete model, impulsive model and timescale model, etc, and studied successfully (see [3, 4, 5]). However, these published results cannot include q-difference Lasota-Wazewska model since the time scale $\overline{q^{\mathbb{Z}}} := \overline{\{q^n : q > 1,\, n \in \mathbb{Z}\}}$ is not periodic time scale under translation. It is known that the q-difference equations are very important dynamic equations which can be applied to model the linear and nonlinear problems and play an important role in different fields of engineering and biological science (see [6, 7]), but there is no result on almost periodic problems of q-difference Lasota-Wazewska model.

In the work [5], the authors proposed a type of impulsive Lasota-Wazewska model with patch structure and forced perturbations to describe the number of red blood cells when blood transfusion is conducted among the animal group. On the other hand, in the aspect of life and engineering sciences, most phenomena are basically modeled as suitable stochastic processes, where relevant parameters are modeled as suitable stochastic processes (see [8, 9, 10, 11, 12]). Based on the background of blood transfusion, we will establish a new neutral impulsive stochastic timescale Lasota-Wazewska model involving q-difference model in this paper. First, by virtue of time scales calculus and new periodic notions of time scales proposed by M. Adıvar, we will introduce two new concepts of mean-square almost periodic stochastic process and consider the following timescale model:

$$\begin{cases}\Delta\big(x_i(t)+c_i(t)x_i(\delta_-(\tau_i,t))\big)=\big[-\alpha_i(t)x_i(t)+\sum\limits_{j=1}^{m}\beta_{ij}(t)e^{-\gamma_{ij}(t)x_j(\delta_-(\tau_{ij},t))}\big]\Delta t\\ \qquad\qquad +\sum\limits_{j=1}^{m}H_{ij}\big(t,x_j(\delta_-(\sigma_{ij},t))\big)\Delta\omega_j(t),\ t\neq t_k,\\ \tilde{\Delta}x_i(t_k)=x_i(t_k^+)-x_i(t_k^+)=I_{ik}(x_i(t_k))+\alpha_{ik}x_i(t_k)+\nu_{ik},\ t=t_k,\end{cases} \tag{1.1}$$

where x_i denotes the number of the red blood cells at time t of the ith animal, $c_i(t)$ is the stimulative rate of the generation of red blood cells per unit time and τ_i is the stimulative time needed to produce blood cells of the ith animal. α_i is the rate of death of the red blood cells of the ith animal, β_{ij} and γ_{ij} describe the generation of red blood cells per unit time and τ_{ij} is the time needed to produce blood cells of the ith animal when blood of the jth animal is transfused into the ith one. $\Delta x_i(t)$ denotes a Δ-stochastic differential of $x_i(t)$, $\alpha_i,\beta_{ij},\gamma_{ij}\in PC_{rd}(\mathbb{T},\mathbb{R}^+)$, $\tau_i,\tau_{ij},\sigma_{ij}$ are some positive constants, $\{t_k\}\in\mathfrak{B}$, $\mathfrak{B}=\big\{\{t_k\}: t_k\in\mathbb{T},\, t_k<t_{k+1},\, k\in\mathbb{Z},\, \lim_{k\to\pm\infty}=\pm\infty\big\}$, the constants $\alpha_{ik},\nu_{ik}\in\mathbb{R}$ and $I_{ik}\in C(L^2(\mathbb{R}),\mathbb{R})$, H_{ij} is Borel measurable, $i=1,2,\dots,n$, $j=1,2,\dots,m$, $k\in\mathbb{Z}$ and $A=(H_{ij})_{n\times m}$ is a diffusion coefficient matrix (i.e., the random perturbation term for the system). The operator $\delta_\pm:\mathbb{T}^*\to\mathbb{T}^*$ are shifts operators satisfying all the conditions in Definition 3 from [13] (here $\overline{\mathbb{T}^*}=\mathbb{T}$, $\overline{\mathbb{T}^*}$ denotes the closure of $\mathbb{T}$, i.e, $\mathbb{T}^*$ is the largest subset of $\mathbb{T}$). Let $(\Omega,\mathbb{F},\mathbb{P})$ be a complete probability space furnished with a complete family of right continuous increasing sub σ-algebras $\{\mathcal{F}_t: t\in[0,+\infty)_{\mathbb{T}}\}$ satisfying $\mathcal{F}_t\subset\mathbb{F}$. $\omega(t)=\big(\omega_1(t),\omega_2(t),\dots,\omega_m(t)\big)$ is an m-dimensional standard

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