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FOV-equivalent block triangular preconditioners for generalized saddle-point problems

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Abstract

We present sufficient conditions for field-of-values-equivalence between block triangular preconditioners and generalized saddle-point matrices arising from inf-sup stable finite element discretizations. We generalize a result by Loghin and Wathen [1] for matrices with a pure saddle-point structure to the case where the $(2,2)$ block is non-zero. Moreover, we extend the analysis to the case where the $(2,1)$ block is not the transpose of the $(1,2)$ block.

Keywords: Preconditioning, Field-of-values-equivalence, Finite Elements, Generalized Saddle-point Problems.

In [1], Loghin and Wathen presented norm-equivalent and field-of-values-equivalent (or FOV-equivalent) block triangular preconditioners for a 2×2 block matrix with a saddle-point structure. FOV-equivalent preconditioners are important in numerical analysis because when the GMRES algorithm is used to solve a linear system $A\mathbf{x} = \mathbf{b}$ preconditioned with a matrix P that is FOV-equivalent to A, the convergence rate does not depend on the size of A ([2, 3, 4, 5]). For finite element applications, this means that the convergence rate is independent on the mesh size and therefore successive refinements of the mesh do not cause a deterioration of the convergence properties. We generalize the result of [1] on FOV-equivalent preconditioners to the case of a 2×2 block matrix where the $(2,2)$ block is non-zero. Moreover, we extend the analysis to the case where the $(2,1)$ block is not the transpose of the $(1,2)$ block. For brevity we only consider the case of right preconditioning. We now introduce some definitions and identities that are used for the rest of the paper. As in [6], $M_{m,n}(\mathbb{R})$ indicates the space of all $m \times n$ matrices with real entries while $M_n(\mathbb{R})$ represents the space of all square matrices of order n with real entries. If $H_1 \in M_n(\mathbb{R})$, $H_2 \in M_m(\mathbb{R})$ are two symmetric positive-definite (SPD) matrices

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