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Note on the Lyapunov functional method[1]

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Abstract. In this note we introduce a direct approach for the Lyapunov functional method to study the global stability of the unique equilibrium solution of reaction diffusion systems and partially degenerate reaction diffusion systems permitting with delays. We first provide the abstract framework and results. Then we give an example as the application.

Keywords: Lyapunov functional method; Global stability; Reaction diffusion systems.

AMS subject classifications (2000): 34C20, 34C25, 92E20.

1 Introduction and the main result

LaSalle Invariance Principle is a powerful stability theorem for dynamic systems and there are several versions. For the finite dimensional dynamic systems, it is relatively simple. For the infinite dimensional dynamic systems, the abstract LaSalle Invariance Principle is not easy to use. In this note we shall introduce a direct approach for the Lyapunov functional method to study the global stability of the unique equilibrium solution of reaction diffusion systems and partially degenerate reaction diffusion systems.

Let $\tau_1, \cdots, \tau_m$ be non-negative constants, $u_1(x,t), \cdots, u_m(x,t)$ be functions of (x,t). We define $\tau = (\tau_1, \cdots, \tau_m)$ and $u^\tau = (u_1(x, t-\tau_1), \cdots, u_m(x, t-\tau_m))$. Let $1 \le k \le m$. We consider the following initial-boundary value problem

$$\begin{cases} u_{it} - d_i \Delta u_i = f_i(x, u, u^\tau), & x \in \Omega, \quad t > 0, & 1 \le i \le k, \\ u_{jt} = f_j(x, u, u^\tau), & x \in \Omega, \quad t > 0, & k+1 \le j \le m, \\ u_i = 0, \quad \text{or} \quad \dfrac{\partial u_i}{\partial \nu} = 0, & x \in \partial\Omega, \quad t > 0, & 1 \le i \le k, \\ u_l(x,t) = u_{l0}(x,t), & x \in \Omega, \quad -\tau_l \le t \le 0, & 1 \le l \le m, \end{cases} \tag{1.1}$$

where $\Omega \subset \mathbb{R}^n$ is a bounded domain with smooth boundary $\partial\Omega$, ν is the outward normal vector of $\partial\Omega$, $d_i > 0$ are constant $(i = 1, \cdots, k)$, $u_{l0}(\cdot, t) \in C^\gamma(\overline{\Omega})$ for some $0 < \gamma < 1$ and uniformly on $-\tau_l \le t \le 0$. If $k < m$, then (1.1) is the initial-boundary value problem of partially degenerate reaction diffusion systems.

Define $d_j = 0$ for $k+1 \le j \le m$. We make the following assumptions:

(H1) The corresponding steady state problem of (1.1)

$$\begin{cases} -d_i \Delta u_i = f_i(x, u, u), & x \in \Omega, \quad 1 \le i \le k, \\ f_j(x, u, u) = 0, & x \in \Omega, \quad k+1 \le j \le m, \\ u_i = 0, \quad \text{or} \quad \dfrac{\partial u_i}{\partial \nu} = 0, & x \in \partial\Omega, \quad 1 \le i \le k \end{cases}$$

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