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Infinitely many solutions for super-quadratic Kirchhoff-type equations with sign-changing potential *

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Abstract: This paper is concerned with the following Kirchhoff type equation

$$\begin{cases} -\left(a+b\int_{\mathbb{R}^3}|\nabla u|^2\mathrm{d}x\right)\triangle u+V(x)u=f(x,u), & x\in\mathbb{R}^3,\\ u(x)\to 0, & |x|\to\infty, \end{cases}$$

where $a>0$, $b\ge 0$, $V\in\mathcal{C}(\mathbb{R}^3,\mathbb{R})$, $f\in\mathcal{C}(\mathbb{R}^3\times\mathbb{R},\mathbb{R})$, $V(x)$ and $f(x,u)u$ are both allowed to be sign-changing. Using a weaker assumption $\lim_{|t|\to\infty}\frac{\int_0^t f(x,s)\mathrm{d}s}{|t|^3}=\infty$ uniformly in $x\in\mathbb{R}^3$, instead of the common assumption $\lim_{|t|\to\infty}\frac{\int_0^t f(x,s)\mathrm{d}s}{|t|^4}=\infty$ uniformly in $x\in\mathbb{R}^3$, we establish the existence of infinitely many high energy solutions of the above problem, where some new tricks are introduced to overcome the competing effect of nonlocal term. Our result unifies both asymptotically cubic or super-cubic cases, which generalizes and improves the existing ones.

Keywords: Kirchhoff type equation; Sign-changing potential; Infinitely many solutions; Super-quadratic growth.

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1 Introduction

In this paper, we study the existence of infinitely many solutions for the Kirchhoff-type equation

$$\begin{cases} -\left(a+b\int_{\mathbb{R}^3}|\nabla u|^2\mathrm{d}x\right)\triangle u+V(x)u=f(x,u), & x\in\mathbb{R}^3,\\ u(x)\to 0, & |x|\to\infty, \end{cases} \tag{1.1}$$

where $a>0$, $b\ge 0$, $V:\mathbb{R}^3\to\mathbb{R}$ and $f:\mathbb{R}^3\times\mathbb{R}\to\mathbb{R}$ satisfy the following assumptions:

(V) $V\in\mathcal{C}(\mathbb{R}^3,\mathbb{R})$, $\inf_{\mathbb{R}^3}V>-\infty$ and there exists a constant $r>0$ such that

$$\lim_{|y|\to\infty}\mathrm{meas}\{x\in\mathbb{R}^3:|x-y|\le r,\ V(x)\le M\}=0,\qquad \forall\ M>0;$$

(F1) $f\in\mathcal{C}(\mathbb{R}^3\times\mathbb{R},\mathbb{R})$ and there exist constants $C_0>0$ and $p\in(2,6)$ such that

$$|f(x,t)|\le C_0\left(|t|+|t|^{p-1}\right),\qquad \forall\ (x,t)\in\mathbb{R}^3\times\mathbb{R};$$

(F2) $\lim_{|t|\to\infty}\frac{F(x,t)}{|t|^3}=\infty$ uniformly in $x\in\mathbb{R}^3$, where $F(x,t):=\int_0^t f(x,s)\mathrm{d}s$;

(F3) there exists a constant $\theta\ge 0$ such that

$$f(x,t)t-4F(x,t)\ge-\theta t^2,\qquad \forall\ (x,t)\in\mathbb{R}^3\times\mathbb{R};$$

(F4) $f(x,t)=-f(x,-t)$ for all $(x,t)\in\mathbb{R}^3\times\mathbb{R}$.

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