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Continuity of the Eigenvalues for a Vibrating Beam

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Abstract

In this paper we prove that the eigenvalues of a vibrating beam have a strongly continuous dependence on the elastic destructive force, i.e., the eigenvalues, as nonlinear functionals of the elastic destructive force, are continuous in the elastic destructive force with respect to the weak topologies in the Lebesgue spaces $\mathcal{L}^p$. In virtue of the minimax characterization for eigenvalues, we prove first the continuity of the lowest eigenvalue and then all the eigenvalues by the induction principle.

Keywords: Eigenvalue, continuity, weak topology.

1. Introduction

For $1 \le p \le \infty$, let $\mathcal{L}^p := L^p([0,1],\mathbb{R})$ be the Lebesgue space with the $\mathcal{L}^p$ norm denoted by $\|\cdot\|_p = \|\cdot\|_{L^p[0,1]}$. From the elementary beam theory, the natural modes of buckling of our problem are the eigenfunctions of a vibrating beam

$$y''''(x) + \lambda y''(x) + F(x)y(x) = 0, \qquad x \in [0,1], \tag{1.1}$$

with the hinged-hinged boundary condition

$$y(0) = y(1) = 0 = y''(0) = y''(1). \tag{1.2}$$

Here, the elastic destructive force (per unit length) $F(x) \in \mathcal{L}^p$ and the eigenvalue λ represents the axial compressive force necessary to cause the beam to buckle. It has been shown in [2] that problem (1.1)-(1.2) has a sequence of real eigenvalues $0 < \lambda_1(F) < \lambda_2(F) < \cdots < \lambda_m(F) < \cdots$ and the nth eigenfunction y_n has $n-1$ zeros η_is interlaced as $0 < \eta_1 < \cdots < \eta_{n-1} < 1$.

In this paper, we will discuss the strongly continuous dependence for the eigenvalues of a vibrating beam on the elastic destructive force. That is, we attempt to prove that as nonlinear functionals of the elastic destructive force, eigenvalues are continuous in the elastic destructive force with respect to the weak topologies in the Lebesgue spaces $\mathcal{L}^p$. This continuity result is the basis to study eigenvalues in a quantitative way. As did in [6, 7, 8, 10] for the second order systems, we will study quantitative analysis for eigenvalues of the fourth order beam equation in future works.

To start with, for the Lebesgue spaces $\mathcal{L}^p$, besides the norm topologies $\|\cdot\|_p$, one can define the weak topologies w_p as follows. We say that $q_n \to q$ in $(\mathcal{L}^p, w_p)$ if $\int_0^1 q_n v \,\mathrm{d}x \to \int_0^1 qv\,\mathrm{d}x$ for each $v \in \mathcal{L}^{p^*}$, where $p^* = p/(p-1)$ is the conjugate exponent of p. Then a functional $f : \mathcal{L}^p \to \mathbb{R}$ is said to be strongly continuous if $f : (\mathcal{L}^p, w_p) \to \mathbb{R}$ is continuous. Based on these two definitions, the main result of this paper on the continuity of $\lambda_m(F)$ in F is described as follows.

Theorem 1.1. For each $m \in \mathbb{N}$, as a nonlinear functional, $\lambda_m(F)$ is strongly continuous in $F \in \mathcal{L}^p$, where $1 \le p \le \infty$.

Since $(\mathcal{L}^1, w_1)$ is the weakest topology, it suffices to show the theorem for the case $p = 1$. Once the Theorem 1.1 is proved, one has that $\lambda_m : (\mathcal{L}^p, \|\cdot\|_p) \to \mathbb{R}, F \to \lambda_m(F)$ is continuous, which means that our continuity conclusion is stronger than the continuity conclusion in [5]. Different from the approaches used in [4, 5, 7, 9] for the second-order equations, we will extensively exploit the minimax characterization for eigenvalues $\lambda_m(F)$s to prove the main result of this paper as follows: Firstly, we show the strongly continuous dependence of the lowest eigenvalue $\lambda_1(F)$ on F in Section 2; Then, we prove the strongly continuous dependence of the higher order eigenvalues $\lambda_m(F)$s $(m > 1)$ by the induction principle in Section 3.

2. The continuity of the lowest eigenvalue

Given $F \in \mathcal{L}^p$, where $1 \le p \le \infty$, and $\lambda \in \mathbb{R}$, let $\varphi_i(x, \lambda, F)$ be the fundamental solution of equation (1.1) satisfying $(y(0), y'(0), y''(0), y'''(0))^T = e_i$, where $1 \le i \le 4$. Results in [4, 9] imply that solutions of (1.1) have a strongly continuous dependence on the elastic destructive force F, i.e., as nonlinear operators, the following solution mappings

$$\mathbb{R} \times (\mathcal{L}^p, w_p) \to (C^3, \|\cdot\|_{C^3}),\ (\lambda, F) \to \varphi_i(\cdot, \lambda, F), \tag{2.1}$$

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