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Existence of nonoscillatory solutions for system of higher-order neutral differential equations with distributed delays

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ABSTRACT

In this paper we consider the existence of nonoscillatory solutions for system of higher-order neutral differential equations with distributed coefficients and delays. We use the *Banach* contraction principle to obtain new sufficient conditions for the existence of nonoscillatory solutions.

1. Introduction and preliminary

In this paper, we consider the system of higher-order neutral differential equations with distributed coefficients and delays

$$\left[r(t)\mathbf{x}(t) + \int_a^b p(t,\theta)\mathbf{x}(t-\theta)d\theta\right]^{(n)} + (-1)^{n+1}[Q_1(t)\mathbf{x}(t-\tau) - Q_2(t)\mathbf{x}(t-\sigma) - \mathbf{h}(t)] = \mathbf{0}, \quad (1)$$

(1) where n is a positive integer, $n \geq 1, 0 < a < b, \tau, \sigma > 0$;
(2) $r \in C\big([t_0,\infty), R^+\big), r(t) > 0, p \in C\big([t_0,\infty) \times [a,b], R\big), \mathbf{h} \in C\big([t_0,\infty), \mathbf{R}^n\big)$;
(3) $\mathbf{x} \in \mathbf{R}^n, Q_i$ is continuous $n \times n$ matrix on $[t_0,\infty), i = 1, 2$.

Recently there has been a lot of activities concerning the existence of nonoscillatory solutions for neutral differential equations with positive and negative coefficients. In 2005, the existence of nonoscillatory solutions

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of first order linear neutral delay differential equations

$$\frac{d}{dt}[x(t)+P(t)x(t-\tau)]+Q_1(t)x(t-\sigma_1)-Q_2(t)x(t-\sigma_2)=0$$

was investigated by Zhang etc. [1]. In 2012, Candan [2] studied higher-order nonlinear differential equation

$$[r(t)[x(t)+P(t)x(t-\tau)]^{(n-1)}]'+(-1)^{n+1}[Q_1(t)g_1(x(t-\sigma_1))-Q_2(t)g_2(x(t-\mu))-f(t)]=0.$$

In 2013, Candan [3] has investigated existence of nonoscillatory solutions for system of higher order nonlinear neutral differential equations

$$[\mathbf{x}(t)+P(t)\mathbf{x}(t-\theta)]^{(n)}+(-1)^{n+1}[Q_1(t)\mathbf{x}(t-\sigma_1)-Q_2(t)\mathbf{x}(t-\sigma_2)]=\mathbf{0}.$$

In same year, Liu etc. [4] have obtained existence of nonoscillatory solutions for system of higher order neutral differential equations

$$[r(t)[\mathbf{x}(t)+P(t)\mathbf{x}(t-\theta)]^{(n-1)}]'+(-1)^n\left(\int_c^d Q_1(t,\tau)\mathbf{x}(t-\tau)d\tau-\int_e^f Q_2(t,\sigma)\mathbf{x}(t-\sigma)d\sigma\right)=\mathbf{0}.$$

As can be seen from the development process of the above equations, the delay of neutral part in the discussed differential equations were all constant delays. However, the case for distributed deviating arguments is rather rare, see [5,6]. In 2015, Candan and Gecgel [6] studied the systems of higher order neutral differential equations with distributed delay

$$\left[\left[x(t)+\int_{a_3}^{b_3}\tilde{P}(t,\xi)x(t-\xi)d\xi\right]\right]'+(-1)^{n+1}\left[\int_{a_1}^{b_1}Q_1(t,\xi)\mathbf{x}(t-\xi)d\xi-\int_{a_2}^{b_2}Q_2(t,\xi)\mathbf{x}(t-\xi)d\xi\right]=\mathbf{0},$$

the discussion only covered the condition for coefficient being $0<\int_{a_3}^{b_3}\tilde{P}(t,\xi)d\xi<\frac{1}{2}$ and $-\frac{1}{2}<\int_{a_3}^{b_3}\tilde{P}(t,\xi)d\xi<0$. However in this paper, the difficulty in establishing feasible operator was settled by skillful use of $r(t)$, the coefficient $\int_a^b p(t,\theta)d\theta$ in the neutral part were all discussed in four cases, that is $(-\infty,-1),(-1,0),(0,1),(1,+\infty)$. Thus, in view of the above, this paper may present its theoretical value as well as practical application value. For related work, we refer the reader to the references [7–10].

A solution of system of equations (1) is a continuous vector function $\mathbf{x}(t)$ defined on $([t_1-\mu,\infty),\mathbf{R}^n)$, for some $t_1>t_0$, such that $r(t)\mathbf{x}(t)+\int_a^b p(t,\theta)\mathbf{x}(t-\theta)d\theta$ is n times continuously differentiable and system of equations (1) holds for all $n\geq 1$. Here, $\mu=\max\{b,\tau,\sigma\}$.

2. The main results

Theorem 1. *Assume that* $0\leq\int_a^b p(t,\theta)d\theta\leq p_1<1$ *and*

$$\int_{t_0}^{\infty}s^{n-1}\|Q_i(s)\|ds<\infty, i=1,2,\int_{t_0}^{\infty}s^{n-1}\|\mathbf{h}(s)\|ds<\infty. \tag{2}$$

Then Eq. (1) has a bounded nonoscillatory solution.

Proof. Let Λ be the set of all continuous and bounded vector functions on $[t_0,\infty)$ with the sup norm. Set $A=\{x\in\Lambda, M_1\leq\|\mathbf{x}(t)\|\leq M_2, t\geq t_0\}$, where M_1,M_2 are two positive constants and $\mathbf{c}$ is a constant vector, such that $p_1M_2+\frac{M_1}{p_1}<\|\mathbf{c}\|<M_2, 1\leq r(t)\leq\frac{1}{p_1}$. From (2), one can choose a $t_1\geq t_0+\mu$, sufficiently