Applied Mathematics Letters 65 (2017) 32–39

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

Asymptotical stability of Riemann–Liouville fractional singular systems with multiple time-varying delays

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ARTICLE INFO

Article history:
Received 22 July 2016
Received in revised form 3 October 2016
Accepted 3 October 2016
Available online 11 October 2016

Keywords:
Asymptotical stability
Riemann–Liouville fractional singular system
Time-varying delay

ABSTRACT

Stability of Riemann–Liouville fractional singular systems remains an open problem. This paper deals with Riemann–Liouville fractional singular systems with multiple time-varying delays, and two asymptotic stability criteria are derived. The criteria are described as matrix equations or matrix inequalities, which are computationally flexible and efficient. The advantage of our employed method is that one may directly calculate integer-order derivatives of the Lyapunov functions. Finally, a simple example is given to illustrate the effectiveness of our main results.

1. Introduction

Fractional calculus and fractional differential systems [1–8] have gained considerable popularity and importance during the past four decades or so, due mainly to its demonstrated applications in widespread fields of science and engineering. The stability of a system is of importance in control theory. A large number of papers [9–14] are devoted to Caputo fractional dynamical systems and various kinds of stability including Mittag-Leffler stability [15,16], asymptotic stability [17,18], uniform stability [19], finite time stability [20] and Ulam stability [21] have been widely studied.

However, the stability of Riemann–Liouville fractional systems makes little progress. In [22], D. Qian et al. study linear Riemann–Liouville fractional differential system with time delays, if all the roots of the characteristic equation have negative real parts, then the trivial solution is asymptotically stable. Liu et al. [23] study Riemann–Liouville fractional systems without and with delay, several sufficient conditions on asymptotical stability are obtained by using Lyapunov direct method.

It should be pointed out that many practical problems are modeled by singular systems [24], such as optimal control problems and constrained control problems, electrical circuits and some population growth

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http://dx.doi.org/10.1016/j.aml.2016.10.002

models. Achieving stability is an important task for system control theory. When the stability of singular systems is compared with nonsingular systems, there are three main difficulties:

(i) It is not easy to satisfy the existence and uniqueness of solutions since the initial conditions may not be consistent;

(ii) It is difficult to calculate the derivatives of Lyapunov functions;

(iii) There often happens impulses in the solutions.

Very recently, Liu et al. [25] investigate stability of fractional singular systems with Caputo derivative, several algebraic criteria are presented in terms of linear matrix inequalities. Motivated by the above research, this paper will deal with the following Riemann–Liouville fractional singular system with multiple time-varying delays:

$$E_{t_0}D_t^q x(t) = Ax(t) + \sum_{i=1}^{m} B_i x(t - \tau_i(t)), \tag{1.1}$$

where $0 < q < 1,\ x(t) \in \mathbf{R}^n$ is the state vector, $E,\ A,\ B_i \in \mathbf{R}^{n\times n},\ i = 1, 2, \ldots, m$ are constant matrices and $0 < \text{rank}E = r < n$. The time-varying delays $\tau_i(t) \ge 0$ are differentiable and $\dot{\tau}_i(t) \le d_i < 1,\ i = 1, 2, \ldots, m$.

Two asymptotic stability criteria are derived of system (1.1). The criteria are described as matrix equations or matrix inequalities, which are computationally flexible and efficient. The advantage of our employed method is that one may directly calculate integer-order derivatives of the Lyapunov functions.

Notations: $\mathbf{R}^n$ denotes an n-dimensional Euclidean space, $\mathbf{R}^{n\times n}$ is the set of all $n\times n$ real matrices, I_n stands for the identity matrix of order n, U^T means the transpose of a real matrix or vector U, $\|x\|$ denotes the Euclidean norm of a real vector x. For a real matrix A, $\lambda_{\max}(A)$ and $\lambda_{\min}(A)$ denote the maximal and the minimal eigenvalue of A, respectively. $\|A\| = \sqrt{\lambda_{\max}(A^TA)}$ denotes the spectral norm of matrix A, $A > 0(\text{or}A < 0)$ means the symmetric matrix A is positive definite (or negative definite).

2. Preliminaries

In this section, some definitions of fractional calculus and important lemmas are introduced.

Definition 2.1 (*[1]*)**.** The Riemann–Liouville fractional integral is defined as

$${}_{t_0}D_t^{-q}x(t) = \frac{1}{\Gamma(q)}\int_{t_0}^{t}(t-s)^{q-1}x(s)ds, \quad (q > 0). \tag{2.1}$$

Definition 2.2 (*[1]*)**.** The Riemann–Liouville fractional derivative is defined as

$${}_{t_0}D_t^{q}x(t) = \frac{1}{\Gamma(n-q)}\frac{d^n}{dt^n}\int_{t_0}^{t}\frac{x(s)}{(t-s)^{q+1-n}}ds, \quad (n-1 \le q < n), \tag{2.2}$$

where Γ denotes the Gamma function.

Property 2.1 (*[2]*)**.** *If $p > q > 0$, then the formulas:*

$${}_{t_0}D_t^{q}({}_{t_0}D_t^{-p}x(t)) = {}_{t_0}D_t^{q-p}x(t). \tag{2.3}$$

hold for "sufficiently good" functions $x(t)$. In particular, this relation holds if $x(t)$ is integrable.

Definition 2.3 (*[25]*)**.** (1) System (1.1) or the pair (E, A) is said to be regular if $\det(s^qE - A) \not\equiv 0$.

(2) System (1.1) or the pair (E, A) is said to be impulse free if $\deg(\det(s^qE - A)) = \text{rank}(E)$.