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Ultimate boundedness theorems for impulsive stochastic differential systems with Markovian switching☆

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ABSTRACT

This paper is concerned with ultimate boundedness analysis of impulsive stochastic differential systems (ISDSs) with Markovian switching. Using the generalized Itô formula, Lyapunov function and matrix inequality technique, several criteria for global pth moment exponential ultimate boundedness are derived. The finding shows that unbounded stochastic differential systems with Markovian switching can be made into bounded systems by impulses.

1. Introduction

ISDSs with Markovian switching are an important class of hybrid dynamical systems which combine the merits of impulsive systems, stochastic systems and Markovian switching systems. Recently, these systems have attracted increasing interest both in theoretical research and applications because they can describe the real systems more reasonably and accurately. In particular, the stability of ISDSs with Markovian switching has been widely studied by many authors (see [1–8]). On the other hand, boundedness is one of the most important and frequently used tools for analyzing the dynamic behavior of dynamical systems, such as stability, attracting and invariant properties, the existence of periodic solution. Therefore boundedness has become a research hot-spot in the study of dynamical systems and many significant progresses have been made in the techniques and methods of investigating the boundedness for many kinds of dynamical systems [9–18].

However, few authors have considered the problem on the boundedness of ISDSs with Markovian switching. This motivates the present study. Using the generalized Itô formula, Lyapunov function and

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matrix inequality technique, several criteria for global pth moment exponential ultimate boundedness are established. The results show that impulses do contribution to the boundedness of stochastic differential systems with Markovian switching even they are unbounded.

2. Preliminaries

Let $\|\cdot\|$ denote the Euclidean norm in $\mathbb{R}^n$, $\mathbb{R}_+ = [0,\infty)$, $\mathbb{R}_{t_0} = [t_0,\infty)$ and $\mathbb{N} = \{1,2,3,\ldots\}$. Let $\mathcal{L}$ denote the well-known $\mathcal{L}$-operator given by the generalized Itô formula (cf. [19,20]). Let $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ denote the minimum and the maximum eigenvalue of the corresponding matrix, respectively. Let $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geq t_0},\mathbb{P})$ be the classical complete probability space with a natural filtration $\{\mathscr{F}_t\}_{t\geq t_0}$ satisfying the usual conditions, and $\omega(t) = (\omega_1(t),\omega_2(t),\ldots,\omega_m(t))^T$ be an m-dimensional Brownian motion defined on the space. Let $r(t)$ be a right continuous Markov chain on $(\Omega,\mathscr{F},\{\mathscr{F}_t\}_{t\geq t_0},\mathbb{P})$ taking values in a finite state space $\Lambda = \{1,2,\ldots,N\}$ with generator $\Gamma = (\gamma_{ij})_{N\times N}$ given by

$$\mathbb{P}\{r(t+\delta) = j|r(t) = i\} = \begin{cases}\gamma_{ij}\delta + o(\delta) & \text{if } i\neq j,\\ 1+\gamma_{ij}\delta + o(\delta) & \text{if } i = j,\end{cases}$$

with $\delta > 0$. Where $\gamma_{ij} \geq 0$ is the transition rate from i to j if $i \neq j$, while $\gamma_{ii} = -\sum_{i\neq j}\gamma_{ij}$. We assume $r(t)$ is independent of $\omega(t)$.

Consider the following ISDS with Markovian switching:

$$\begin{cases} dx(t) = f(x(t),t,r(t))dt + g(x(t),t,r(t))d\omega(t),\ t\neq t_k, t\geq t_0,\\ \Delta x(t_k) = I_k(x(t_k^-),t_k),\ k\in\mathbb{N},\\ x(t_0) = x_0,\end{cases} \tag{1}$$

where $f : \mathbb{R}^n\times\mathbb{R}_{t_0}\times\Lambda \to \mathbb{R}^n$, $g : \mathbb{R}^n\times\mathbb{R}_{t_0}\times\Lambda \to \mathbb{R}^{n\times m}$, $I_k : \mathbb{R}^n\times\mathbb{R}_{t_0} \to \mathbb{R}^n$, and $\{t_k, k\in\mathbb{N}\}$ is a strictly increasing sequence which satisfies $\lim_{k\to\infty} t_k = \infty$.

In this paper, we assume that for any $x_0 \in \mathbb{R}^n$, Sys. (1) has at least one solution $x(t)$. For results on the existence–uniqueness of ISDSs with Markovian switching, one may refer to [21].

Definition 2.1. Sys. (1) is said to be globally pth moment exponentially ultimately bounded (EUB) with the exponential convergence rate λ and the ultimate bound M if there exist constants $\lambda > 0$, $K > 0$ and $M \geq 0$ such that for all $x_0 \in \mathbb{R}^n$,

$$\mathbb{E}\|x(t)\|^p \leq K\mathbb{E}\|x_0\|^p e^{-\lambda(t-t_0)} + M,\ p > 0, t\geq t_0. \tag{2}$$

Especially, Sys. (1) is said to be globally pth moment exponentially stable (ES) with the exponential convergence rate λ when $M = 0$. When $p = 2$, globally pth moment EUB (ES) is often called to be globally EUB (ES) in mean square.

Lemma 2.1 ([22]). *Let $Y \in \mathbb{R}^{n\times n}$ be a positive definite matrix and $Q \in \mathbb{R}^{n\times n}$ a symmetric matrix. For $x\in\mathbb{R}^n$, $\lambda_{\min}(Y^{-1}Q)\cdot x^TYx \leq x^TQx \leq \lambda_{\max}(Y^{-1}Q)x^TYx$.*

Lemma 2.2 ([23]). *For $x_i \geq 0$, $\alpha_i > 0$ and $\sum_{i=1}^n \alpha_i = 1$, $\prod_{i=1}^n x_i^{\alpha_i} \leq \sum_{i=1}^n \alpha_i x_i$.*

3. Main results

Theorem 3.1. *Assume that there exist symmetric positive-definite matrices Q_i and locally integrable nonnegative functions $\xi_i(t)$ and functions $K_1(t)$, $K_2(t)$, $J_1(t)$ and $J_2(t)$ and constants $\beta_k > 0$, $\alpha < 0$, $\kappa_1 \in \mathbb{R}$ and $\kappa_2 \in \mathbb{R}_+$ such that*