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Nontrivial solutions for a fractional advection dispersion equation in anomalous diffusion

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Abstract. In this paper, we consider the existence of nontrivial solutions for a class of fractional advection-dispersion equations. A new existence result is established by introducing a suitable fractional derivative Sobolev space and using the critical point theorem.
Keywords. Fractional advection-dispersion equation; Critical point theorem; Anomalous diffusion; Variational methods.

1 Introduction

In [1], Risken introduced an advection-dispersion equation to describe the Brownian motion of particles

$$\frac{\partial C(x,t)}{\partial t} = \left[-v\frac{\partial}{\partial x} + D\frac{\partial^2}{\partial x^2}\right] C(x,t),$$

where $C(x,t)$ is a concentration field of space variable x at time t, $D > 0$ is the diffusion coefficient and $v > 0$ is the drift coefficient. Many laboratory data [2, 3, 4] and numerical experiments [5, 6] indicate that solutes moving through a highly heterogeneous aquifer violate the basic assumptions of the local second order theories because of the large deviations due to the stochastic process of Brownian motion. According to [2], an anomalous dispersion process should be described by the following advection-dispersion equation containing the left and the right fractional differential operators

$$\frac{\partial C(x,t)}{\partial t} = -v\frac{\partial C(x,t)}{\partial x} + Dj\frac{\partial^\gamma C(x,t)}{\partial x^\gamma} + D(1-j)\frac{\partial^\gamma C(x,t)}{\partial(-x)^\gamma}, \tag{1.1}$$

where C is the expected concentration field of space variable x at time t, v is a constant mean velocity, x is the distance in the direction of the mean velocity, D is a constant dispersion coefficient, $0 \le j \le 1$ describes the skewness of the transport process, and γ is the order of left and right fractional differential operators. Especially, if $\gamma = 2$, the dispersion operator reduces to the classical advection-dispersion operator and (1.1) becomes the classical advection-dispersion equation. On the other hand, if $j = \frac{1}{2}$, (1.1) describes symmetric transitions. Define an equivalent Riesz potential symmetric operator [7]

$$2\nabla^\gamma \equiv D_+^\gamma + D_-^\gamma,$$

which gives the mass balance equation for the symmetric fractional advection dispersion

$$\frac{\partial C(x,t)}{\partial t} = -v\nabla C(x,t) + D\nabla^\gamma C(x,t). \tag{1.2}$$

For a one dimensional symmetric case, by using Ekeland's variational principle and the mountain pass theorem, Jiao and Zhou [8] established the existence of at least one nonzero solution for the following equation

$$\begin{cases} \dfrac{d}{dt}\left(\dfrac{1}{2}{}_0D_t^{-\beta}(u'(t)) + \dfrac{1}{2}{}_tD_T^{-\beta}(u'(t))\right) + \nabla F(t,u(t)) = 0, \text{ a.e. } t \in [0,T], \\ u(0) = u(T) = 0, \end{cases} \tag{1.3}$$

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