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Ground state solutions to biharmonic equations involving critical nonlinearities and multiple singular potentials

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Abstract

In this paper, a biharmonic problem is investigated, which involves critical Sobolev nonlinearity and multiple Rellich–type terms. By complicated asymptotic analysis and variational arguments, the existence and nonexistence of ground state solution to the problem are established.

Keywords: Biharmonic problem; ground state solution; Rellich inequality; variational method.
2000 Mathematics Subject Classification: 35J60, 31B30, 35B33, 35B25

1 Introduction

In this paper, we study the following biharmonic problem:

$$\begin{cases} \Delta^2 u - \sum_{i=1}^{k} \frac{\mu_i}{|x-a_i|^4} u = u^{2^*-1}, \\ u \in D^{2,2}(\mathbb{R}^N), \quad u \geq 0 \text{ in } \mathbb{R}^N \setminus \{a_1, a_2, \cdots, a_k\}, \end{cases} \tag{1.1}$$

where $2^* := \frac{2N}{N-4}$ is the critical Sobolev exponent, $D^{2,2}(\mathbb{R}^N)$ is the completion of $C_0^\infty(\mathbb{R}^N)$ with respect to $(\int_{\mathbb{R}^N} |\Delta \cdot|^2 \,\mathrm{d}x)^{1/2}$ and the parameters satisfy the assumption:

$$(\mathcal{H}_1) \quad N \geq 5, \quad k \geq 2, \quad \mu_i \in \mathbb{R}, \quad a_i \in \mathbb{R}^N, \ a_i \neq a_j, \ i \neq j, \ 1 \leq i, j \leq k.$$

Problem (1.1) is related to the following Rellich inequality [1]:

$$\int_{\mathbb{R}^N} \frac{u^2}{|x-a|^4} \mathrm{d}x \leq \frac{1}{\bar{\mu}} \int_{\mathbb{R}^N} |\Delta u|^2 \mathrm{d}x, \quad \forall a \in \mathbb{R}^N, \ u \in D^{2,2}(\mathbb{R}^N),$$

where $\bar{\mu} := \left(\frac{N(N-4)}{4}\right)^2$. Then the following best Sobolev constant is well defined:

$$S(\mu) = \inf_{u \in D^{2,2}(\mathbb{R}^N) \setminus \{0\}} \frac{\int_{\mathbb{R}^N} \left(|\Delta u|^2 - \mu \frac{u^2}{|x-a|^4}\right) \mathrm{d}x}{\left(\int_{\mathbb{R}^N} |u|^{2^*} \mathrm{d}x\right)^{\frac{2}{2^*}}}, \quad a \in \mathbb{R}^N, \ \mu < \bar{\mu}.$$

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