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# Ground state solutions to biharmonic equations involving critical nonlinearities and multiple singular potentials

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## Abstract

In this paper, a biharmonic problem is investigated, which involves critical Sobolev nonlinearity and multiple Rellich-type terms. By complicated asymptotic analysis and variational arguments, the existence and nonexistence of ground state solution to the problem are established.

**Keywords:** Biharmonic problem; ground state solution; Rellich inequality; variational method.

**2000 Mathematics Subject Classification:** 35J60, 31B30, 35B33, 35B25

## 1 Introduction

In this paper, we study the following biharmonic problem:

$$\begin{cases} \Delta^2 u - \sum_{i=1}^k \frac{\mu_i}{|x - a_i|^4} u = u^{2^*-1}, \\ u \in D^{2,2}(\mathbb{R}^N), \quad u \geq 0 \text{ in } \mathbb{R}^N \setminus \{a_1, a_2, \dots, a_k\}, \end{cases} \quad (1.1)$$

where  $2^* := \frac{2N}{N-4}$  is the critical Sobolev exponent,  $D^{2,2}(\mathbb{R}^N)$  is the completion of  $C_0^\infty(\mathbb{R}^N)$  with respect to  $(\int_{\mathbb{R}^N} |\Delta \cdot|^2 dx)^{1/2}$  and the parameters satisfy the assumption:

$$(\mathcal{H}_1) \quad N \geq 5, \quad k \geq 2, \quad \mu_i \in \mathbb{R}, \quad a_i \in \mathbb{R}^N, \quad a_i \neq a_j, \quad i \neq j, \quad 1 \leq i, j \leq k.$$

Problem (1.1) is related to the following Rellich inequality [1]:

$$\int_{\mathbb{R}^N} \frac{u^2}{|x - a|^4} dx \leq \frac{1}{\bar{\mu}} \int_{\mathbb{R}^N} |\Delta u|^2 dx, \quad \forall a \in \mathbb{R}^N, \quad u \in D^{2,2}(\mathbb{R}^N),$$

where  $\bar{\mu} := (\frac{N(N-4)}{4})^2$ . Then the following best Sobolev constant is well defined:

$$S(\mu) = \inf_{u \in D^{2,2}(\mathbb{R}^N) \setminus \{0\}} \frac{\int_{\mathbb{R}^N} (|\Delta u|^2 - \mu \frac{u^2}{|x-a|^4}) dx}{(\int_{\mathbb{R}^N} |u|^{2^*} dx)^{\frac{2}{2^*}}}, \quad a \in \mathbb{R}^N, \quad \mu < \bar{\mu}.$$

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