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To the question of efficiency of iterative methods

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ABSTRACT

In this paper we prove that, among all one-point iterative processes without memory of order p, the most efficient processes are of order $p = 3$. Moreover, the computational efficiency of one-point iterative processes without memory decreases to 1 as p increases, i.e., the efficiency index of higher order of convergence methods is low. We find the upper and lower bounds of the Ostrowski–Traub index of computational efficiency in a wider class of iterative methods with unit informational efficiency.

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1. Introduction

Let α be a simple real root of a single non-linear equation of the form

$$f(x) = 0, \tag{1}$$

and x_0 be an initial approximation to α.

For solving (1) we apply a one-point iterative method without memory

$$x_n = F(x_{n-1}), \qquad n = 1, 2, \ \ldots \tag{2}$$

where $F(\alpha) = \alpha$.

Recall that, according to the classification of Traub [1], a one-point iterative method without memory is determined, at each step n, only by the new information at x_{n-1}, and we do not need to remember any evaluations from previous steps. Many classical methods, such as

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Newton's method, of order 2:

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})}, \qquad n = 1, 2, \ \dots, \tag{3}$$

Chebyshev's method, of order 3:

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})} \cdot \left(1 + \frac{f(x_{n-1})f''(x_{n-1})}{2\big(f'(x_{n-1})\big)^2}\right), \tag{4}$$

and

Halley's method [2], of order 3:

$$x_n = x_{n-1} - \frac{f(x_{n-1})}{f'(x_{n-1})} \cdot \frac{1}{1 - \frac{f(x_{n-1})f''(x_{n-1})}{2\left(f'(x_{n-1})\right)^2}}, \tag{5}$$

belong to this category.

In this paper we obtain bounds for the efficiency index of the iterative methods of a certain class and identify the optimal order among these methods.

The paper is organized as follows: Section 2 contains the notations, definitions and some previous results, pertaining to this paper. Section 3 provides our main results, where we obtain the tight bounds for the efficiency index of one-point methods without memory and prove that the index is maximal for processes of order 3. Section 4 contains brief conclusions.

2. Definitions and notations

2.1. Order of convergence

Let $e_n = x_n - \alpha$ be the *error* at the nth stage of a convergent iterative process. If

$$|e_{n+1}| = c|e_n|^k + o(|e_n|^k), \tag{6}$$

where c and k are some positive constants, then the iterative process *converges with order* $p = k$, and the number c is called the *asymptotic error constant*. In 1870, Schröder [3] defined the order of convergence as follows: an iterative function F is of order p if

$$F(\alpha) = \alpha, \qquad F^{(i)}(\alpha) = 0, \ \ 1 \le i \le p-1, \qquad F^{(p)}(\alpha) \ne 0.$$

Clearly, this definition is valid only for integer values of order p and for iterative functions with p continuous derivatives.

Wall [4] defined the order of the process in a more general way, namely by

$$p = \lim_{n\to\infty} \frac{p_{n+1}}{p_n}, \tag{7}$$

where $p_n = -\log|e_n|$, if the limit of the right-hand side exists. Note that Definition (7) generalizes (6). Indeed, if the process converges with order k (i.e., (6) takes place) then, for sufficiently large n,

$$p_{n+1} \approx -\log c + kp_n,$$

or, equivalently,

$$\frac{p_{n+1}}{p_n} \approx -\frac{\log c}{p_n} + k.$$

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