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Existence of non-trivial solutions for nonlinear fractional Schrödinger–Poisson equations

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A R T I C L E I N F O

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A B S T R A C T

We study the nonlinear fractional Schrödinger–Poisson equation

$$\begin{cases} (-\Delta)^s u + u + \phi u = f(x,u), & \text{in } \mathbb{R}^3, \\ (-\Delta)^t \phi = u^2, & \text{in } \mathbb{R}^3, \end{cases}$$

where $s, t \in (0,1]$, $2t + 4s > 3$. Under some assumptions on f, we obtain the existence of non-trivial solutions. The proof is based on the perturbation method and the mountain pass theorem.

1. Introduction

In this paper, we are concerned with the existence of non-trivial solutions for the following fractional Schrödinger–Poisson equation

$$\begin{cases} (-\Delta)^s u + u + \phi u = f(x,u), & \text{in } \mathbb{R}^3, \\ (-\Delta)^t \phi = u^2, & \text{in } \mathbb{R}^3, \end{cases} \tag{1.1}$$

where $s, t \in (0,1]$, $2t + 4s > 3$, $(-\Delta)^s$ denotes the fractional Laplacian.

When $s = t = 1$, Eq. (1.1) reduces to a Schrödinger–Poisson equation, which describes system of identical charged particles interacting each other in the case where magnetic effects can be neglected [1,2]. If we only consider the first equation in (1.1) and assume that $\phi = 0$, then it reduces to a fractional Schrödinger equation, which is a fundamental equation in fractional quantum mechanics [3,4].

Fractional Schrödinger–Poisson equations have attracted some attention in recent years. Wei [5] studied the existence of infinitely many solutions for the following equation

$$\begin{cases} (-\Delta)^s u + V(x)u + \phi u = f(x,u), & \text{in } \mathbb{R}^3, \\ (-\Delta)^s \phi = \gamma_\alpha u^2, & \text{in } \mathbb{R}^3, \end{cases} \tag{1.2}$$

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where $s \in (0,1]$, γ_α is a positive constant. Teng [6] studied the existence of ground state solutions for the system

$$\begin{cases}(-\Delta)^s u + V(x)u + \phi u = \mu|u|^{q-1}u + |u|^{2_s^*-2}u, & \text{in } \mathbb{R}^3,\\ (-\Delta)^t\phi = u^2, & \text{in } \mathbb{R}^3,\end{cases} \tag{1.3}$$

where $\mu \geq 0$, $1 < q < 2_s^* - 1 = \frac{3+2s}{3-2s}$, $s, t \in (0,1)$, $2t + 2s > 3$. Zhang [7] considered the existence of positive solutions for the equation

$$\begin{cases}(-\Delta)^s u + \lambda\phi u = g(u), & \text{in } \mathbb{R}^3,\\ (-\Delta)^t\phi = \lambda u^2, & \text{in } \mathbb{R}^3,\end{cases} \tag{1.4}$$

where $s, t \in [0,1]$, g is a nonlinearity of Berestycki–Lions type.

Recently, some authors proposed a new approach called perturbation method to study the quasilinear elliptic equations, see [8]. The idea is to get the existence of critical points of the perturbed energy functional I_λ for $\lambda > 0$ small and then taking $\lambda \to 0$ to obtain solutions of original problems. Very recently, Feng [9] used the perturbation method to study the Schrödinger–Poisson equation

$$\begin{cases}-\Delta u + u + \phi u = f(x,u), & \text{in } \mathbb{R}^3,\\ (-\Delta)^{\alpha/2}\phi = u^2, \quad \lim\limits_{|x|\to\infty}\phi(x) = 0, & \text{in } \mathbb{R}^3,\end{cases} \tag{1.5}$$

where $\alpha \in (1,2]$. Under some conditions, the problem (1.5) possesses at least a nontrivial solution.

We point out that when $s = 1$ and $t \in (\frac{1}{2}, 1]$, the problem (1.1) boils down to (1.5). From the assumptions about the potential V in [5], it is obvious that V is not a constant, we cannot use the similar approach as in [5] to study the problem (1.1). In [7], the nonlinear term g in problem (1.4) is required to be $C^1(\mathbb{R},\mathbb{R})$, we do not need this condition here since the perturbed method is exploited. The main result of this paper is described as follows.

Theorem 1.1. *Suppose f satisfies the following conditions:*
(A1) *$f \in C(\mathbb{R}^3 \times \mathbb{R}, \mathbb{R})$, for every $x \in \mathbb{R}^3$ and $u \in \mathbb{R}$, there exist constants $C_1 > 0$ and $p \in (2, 2_s^*)$ such that*

$$|f(x,u)| \leq C_1(|u| + |u|^{p-1}),$$

where $2_s^* = \frac{6}{3-2s}$ is the fractional critical Sobolev exponent;
(A2) *$f(x,u) = o(|u|)$, $|u| \to 0$, uniformly on $\mathbb{R}^3$;*
(A3) *there exists $\mu > 4$ such that*

$$0 < \mu F(x,u) \leq uf(x,u)$$

holds for every $x \in \mathbb{R}^3$ and $u \in \mathbb{R} \setminus \{0\}$, where $F(x,u) = \int_0^u f(x,s)ds$;
Then problem (1.1) has at least a nontrivial solution.

The paper is organized as follows. In Section 2, we present some preliminary results. In Section 3, we will prove Theorem 1.1.

2. Preliminaries

For $p \in [1,\infty)$, we denote by $L^p(\mathbb{R}^3)$ the usual Lebesgue space with the norm $\|u\|_p = \left(\int_{\mathbb{R}^3}|u|^p dx\right)^{\frac{1}{p}}$. We recall some definitions of fractional Sobolev spaces and the fractional Laplacian, for more details, we refer to [10]. The fractional Sobolev space $H^s(\mathbb{R}^3)$ is defined as follows

$$H^s(\mathbb{R}^3) = \left\{u \in L^2(\mathbb{R}^3) : \int_{\mathbb{R}^3}(1 + |\xi|^{2s})|\mathcal{F}u(\xi)|^2 d\xi < \infty\right\}$$