Accepted Manuscript

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PII: S0893-9659(17)30115-5
DOI: http://dx.doi.org/10.1016/j.aml.2017.04.002
Reference: AML 5223

To appear in: *Applied Mathematics Letters*

Received date : 10 February 2017
Revised date : 1 April 2017
Accepted date : 1 April 2017

Please cite this article as: S. Filipov, et al., Shooting-projection method for two-point boundary value problems, Appl. Math. Lett. (2017), http://dx.doi.org/10.1016/j.aml.2017.04.002

Shooting-Projection Method for Two-Point Boundary Value Problems

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Keywords

Two-point boundary value problem, shooting-projection method, H^1 semi-norm

Abstract

This paper presents a novel shooting method for solving two-point boundary value problems for second order ordinary differential equations. The method works as follows: first, a guess for the initial condition is made and an integration of the differential equation is performed to obtain an initial value problem solution; then, the end value of the solution is used in a simple iteration formula to correct the initial condition; the process is repeated until the second boundary condition is satisfied. The iteration formula is derived utilizing an auxiliary function that satisfies both boundary conditions and minimizes the H^1 semi-norm of the difference between itself and the initial value problem solution.

Introduction

Let $u(t)$ be a real-valued function of a real independent variable $t\in[a,b]$. Consider the two-point boundary value problem (TPBVP) [1], [2], [3]

$$u''=f(t,u,u'),\ t\in(a,b), \tag{1}$$
$$u(a)=u_a,\ u(b)=u_b, \tag{2}$$

where a, b, u_a, and u_b are given constants, and f is a given function that specifies the differential equation (1). Let us assume that f is such that the problem (1),(2) has a unique solution on the interval $[a,b]$. The basic idea of any shooting method for solving TPBVPs is to replace the boundary conditions (2) with the initial conditions

$$u(a)=u_a,\ u'(a)=V_a, \tag{3}$$

and treat the TPBVP as an initial value problem (IVP). In (3) V_a is the value of the derivative of the TPBVP solution at the first (left) boundary $t=a$. Since V_a is not known, one can make a guess for its value and solve (1) together with (3), using the guess value v_a instead of V_a. The obtained IVP solution $u(t;v_a)$ satisfies $u'(a;v_a)=v_a$ and the first boundary condition, i.e. $u(a;v_a)=u_a$, but typically it does not satisfy the second (right) boundary condition, i.e. $u(b;v_a)\neq u_b$. The difference

$$E(v_a)=u(b;v_a)-u_b \tag{4}$$

is the *deviation* from the second boundary condition. Due to the uniqueness of the TPBVP solution, $E(v_a)=0$ if and only if $v_a=V_a$. Then, the corresponding IVP solution $u(t;V_a)$ is the sought TPBVP