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DIFFERENTIAL EQUATIONS WITH SEVERAL NON-MONOTONE ARGUMENTS: AN OSCILLATION RESULT

G. E. CHATZARAKIS AND H. PÉICS

ABSTRACT. This paper is concerned with the oscillatory behavior of first-order differential equations with several non-monotone delay arguments and nonnegative coefficients. A sufficient condition, involving limsup, which guarantees the oscillation of all solutions is established. Also, an example illustrating the significance of the result is given.

Keywords: differential equation, non-monotone argument, oscillatory solutions, nonoscillatory solutions.

2010 Mathematics Subject Classification: 34K11, 34K06.

1. INTRODUCTION

In this paper we consider the differential equation with several non-monotone delay arguments

$$x'(t) + \sum_{i=1}^{m} p_i(t) x\left(\tau_i(t)\right) = 0, \quad t \ge 0,$$
(1.1)

where p_i , $1 \le i \le m$, are functions of nonnegative real numbers, and τ_i , $1 \le i \le m$, are Lebesgue measurable functions satisfying

$$\tau_i(t) < t, \quad t \ge 0 \quad \text{and} \quad \lim_{t \to \infty} \tau_i(t) = \infty, \quad 1 \le i \le m.$$
 (1.2)

In addition, we consider the initial condition for (1.1)

$$x(t) = \varphi(t), \quad t \le 0, \tag{1.3}$$

where $\varphi: (-\infty, 0] \to \mathbb{R}$ is a bounded Borel measurable function.

A solution of (1.1), (1.3) is an absolutely continuous on $[0, \infty)$ function satisfying (1.1) for all $t \ge 0$ and (1.3) for all $t \le 0$.

A solution x(t) of (1.1) is oscillatory, if it is neither eventually positive nor eventually negative. If there exists an eventually positive or an eventually negative solution, the equation is *nonoscillatory*. An equation is *oscillatory* if all its solutions oscillate.

The problem of establishing sufficient conditions for the oscillation of all solutions of equation (1.1) has been the subject of many investigations. See, for example, [1-3, 5-12] and the references cited therein. For the general oscillation theory of differential equations the reader is referred to the monograph [4].

In 1978 Ladde [10] and in 1982 Ladas and Stavroulakis [9] proved that if

$$\liminf_{t \to \infty} \int_{\tau(t)}^{t} \sum_{i=1}^{m} p_i(s) ds > \frac{1}{e}, \tag{1.4}$$

where $\tau(t) = \max_{1 \le i \le m} \{\tau_i(t)\}$, then all solutions of (1.1) oscillate.

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