Accepted Manuscript

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PII: S0893-9659(16)30356-1
DOI: http://dx.doi.org/10.1016/j.aml.2016.12.005
Reference: AML 5142

To appear in: *Applied Mathematics Letters*

Received date : 26 October 2016
Revised date : 4 December 2016
Accepted date : 4 December 2016

DIFFERENTIAL EQUATIONS WITH SEVERAL NON-MONOTONE ARGUMENTS: AN OSCILLATION RESULT

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ABSTRACT. This paper is concerned with the oscillatory behavior of first-order differential equations with several non-monotone delay arguments and non-negative coefficients. A sufficient condition, involving lim sup, which guarantees the oscillation of all solutions is established. Also, an example illustrating the significance of the result is given.

Keywords: differential equation, non-monotone argument, oscillatory solutions, nonoscillatory solutions.

2010 Mathematics Subject Classification: 34K11, 34K06.

1. INTRODUCTION

In this paper we consider the differential equation with several non-monotone delay arguments

$$x'(t) + \sum_{i=1}^{m} p_i(t)x\left(\tau_i(t)\right) = 0, \quad t \geq 0, \tag{1.1}$$

where p_i, $1 \leq i \leq m$, are functions of nonnegative real numbers, and τ_i, $1 \leq i \leq m$, are Lebesgue measurable functions satisfying

$$\tau_i(t) < t, \quad t \geq 0 \qquad \text{and} \qquad \lim_{t\to\infty} \tau_i(t) = \infty, \quad 1 \leq i \leq m. \tag{1.2}$$

In addition, we consider the initial condition for (1.1)

$$x(t) = \varphi(t), \ \ t \leq 0, \tag{1.3}$$

where $\varphi : (-\infty, 0] \to \mathbb{R}$ is a bounded Borel measurable function.

A solution of (1.1), (1.3) is an absolutely continuous on $[0, \infty)$ function satisfying (1.1) for all $t \geq 0$ and (1.3) for all $t \leq 0$.

A solution $x(t)$ of (1.1) is *oscillatory*, if it is neither eventually positive nor eventually negative. If there exists an eventually positive or an eventually negative solution, the equation is *nonoscillatory*. An equation is *oscillatory* if all its solutions oscillate.

The problem of establishing sufficient conditions for the oscillation of all solutions of equation (1.1) has been the subject of many investigations. See, for example, [1−3, 5−12] and the references cited therein. For the general oscillation theory of differential equations the reader is referred to the monograph [4].

In 1978 Ladde [10] and in 1982 Ladas and Stavroulakis [9] proved that if

$$\liminf_{t\to\infty} \int_{\tau(t)}^{t} \sum_{i=1}^{m} p_i(s)ds > \frac{1}{e}, \tag{1.4}$$

where $\tau(t) = \max_{1\leq i\leq m}\{\tau_i(t)\}$, then all solutions of (1.1) oscillate.