On symplectic transformations of linear Hamiltonian differential systems without normality

Julia Elyseeva

PII: S0893-9659(16)30379-2
DOI: http://dx.doi.org/10.1016/j.aml.2016.12.012
Reference: AML 5149

To appear in: *Applied Mathematics Letters*

Received date : 8 November 2016
Revised date : 22 December 2016
Accepted date : 22 December 2016

On symplectic transformations of linear Hamiltonian differential systems without normality

Julia Elyseeva

Department of Applied Mathematics, Moscow State University of Technology, Vadkovskii per. 3a, 101472, Moscow, Russia
e-mail:elyseeva@mtu-net.ru

Abstract

In this paper we investigate oscillations of conjoined bases of linear Hamiltonian differential systems related via symplectic transformations. Both systems are considered without controllability (or normality) assumptions and under the Legendre condition for their Hamiltonians. The main result of the paper presents new explicit relations connecting the multiplicities of proper focal points of $Y(t)$ and the transformed conjoined basis $\tilde{Y}(t) = R^{-1}(t)Y(t)$, where the symplectic transformation matrix $R(t)$ obeys some additional assumptions on the rank of its components. As consequences of the main result we formulate the generalized reciprocity principle for the Hamiltonian systems without normality. The main tool of the paper is the comparative index theory for discrete symplectic systems generalized to the continuous case.

Keywords: Linear Hamiltonian differential systems, Transformation theory, Reciprocity principle, Comparative index
2000 MSC: 34C10

1. Introduction

The principal concern of this paper is the mutual oscillatory behaviour of the linear Hamiltonian systems [1]

$$y'(t) = J\mathcal{H}(t)y(t),\ \mathcal{H}(t) = \begin{bmatrix} -C(t) & A^T(t) \\ A(t) & B(t) \end{bmatrix},\ \mathcal{H}(t) = \mathcal{H}^T(t),\ J = \begin{bmatrix} 0 & I \\ -I & 0 \end{bmatrix},\ t \in [a, \infty) \tag{1.1}$$

and

$$\tilde{y}'(t) = J\tilde{\mathcal{H}}(t)\tilde{y}(t),\ \tilde{\mathcal{H}}(t) = \begin{bmatrix} -\tilde{C}(t) & \tilde{A}^T(t) \\ \tilde{A}(t) & \tilde{B}(t) \end{bmatrix},\ \tilde{\mathcal{H}}(t) = \tilde{\mathcal{H}}^T(t),\ t \in [a, \infty) \tag{1.2}$$

which are related via the symplectic transformation

$$\tilde{y}(t) = R^{-1}(t)y(t), \quad R(t) = \begin{bmatrix} L(t) & M(t) \\ K(t) & N(t) \end{bmatrix}, \quad R^T(t)JR(t) = J,\ t \in [a, \infty). \tag{1.3}$$

Here we assume that the $n \times n$ blocks of $\mathcal{H}(t)$, $\tilde{\mathcal{H}}(t)$ are real piecewise continuous matrix functions of t and we use the notation $I,\ 0$ for the identity and zero matrices. It is well known [4] that if $R(t) \in \mathbb{R}^{2n\times 2n}$ is symplectic, then (1.2) is also a linear Hamiltonian system, in more details, $\tilde{\mathcal{H}}(t) = R^T(t)(H(t) - J^T R'(t)R^{-1}(t))R(t) = \tilde{\mathcal{H}}^T(t)$, where $J^T R'(t)R^{-1}(t) = JR'(t)JR^T(t)J$ is symmetric by (1.3). We assume that $R(t) \in C^1_p$ (i.e., $R(t)$ is continuous with piecewise continuous $R'(t)$) and that systems (1.1), (1.2) obey the so-called Legendre conditions

$$B(t) \geq 0,\ t \in t \in [a, \infty), \tag{1.4}$$
$$\tilde{B}(t) \geq 0,\ t \in t \in [a, \infty), \tag{1.5}$$

where $A \geq 0$ means that the symmetric matrix A is nonnegative definite.

In the classical transformation theory, such as in [2, 3, 4, 5, 6] systems (1.1), (1.2) are studied under controllability (or normality) assumption, see [7, Section 4.1]. Using the controllability assumption and (1.4), (1.5) it has been shown in [2, 4, 8] that for $R(t) = J$ system (1.1) is nonoscillatory if and only if the so-called *reciprocal* system (1.2) with the Hamiltonian $\tilde{\mathcal{H}}(t) = J^T\mathcal{H}(t)J$ is nonoscillatory. This statement is now commonly referred as *reciprocity principle* for Hamiltonian systems. In [5] the reciprocity-type statement was extended under natural additional assumptions to general transformation (1.3). For example, it is proved in [5, Theorem 1], that under the assumption