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Limit Sets within Curves Where Trajectories Converge to

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Abstract

For continuously differentiable vector fields, we characterize the ω limit set of a trajectory converging to a compact curve $\Gamma \subset \mathbb{R}^n$. In particular, the limit set is either a fixed point or a continuum of fixed points if Γ is a simple open curve; otherwise, the limit set can in addition be either a closed orbit or a number of fixed points with compatibly oriented orbits connecting them. An implication of the result is a tightened-up version of the Poincaré-Bendixson theorem.

Keywords: limit set, convergence, curve, Poincaré-Bendixson Theorem

1. Introduction

Convergence of trajectories is a fundamental problem in dynamical systems [8]. Understanding how the solutions of a system converge, if they converge, provides qualitative information on the transient and steady-state behavior of the system. Such studies are particularly relevant for the qualitative description of population dynamics [11, 10, 9], where a limit set has the interpretation as those individuals that survive in the long run. Related studies can also be found in slow-fast systems [6] where usually trajectories converge to the lower-dimensional slow manifold. Most of the available research focuses on the stability of an invariant set [3, 13, 4]. Examples of fundamental results in such a setting are Lyapunov's asymptotic stability and LaSalle's invariance principle [12]. In particular, for the latter, even if it is known that a certain invariant set is, e.g. asymptotically stable, it is not straightforward to further conclude precisely to where the trajectories in the neighborhood of such a set converge.

We address exactly such a problem in the case when a trajectory converges to a compact curve. More specifically, given an n-dimensional vector field, $n \geq 2$, having a one-dimensional invariant set Γ, we show that if a trajectory converges to Γ, then the omega limit set of the trajectory is either