Applied Mathematics Letters 68 (2017) 109–116

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

A hybrid-mesh hybridizable discontinuous Galerkin method for solving the time-harmonic Maxwell's equations

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A R T I C L E   I N F O

Article history:
Received 22 October 2016
Received in revised form 28 December 2016
Accepted 29 December 2016
Available online 16 January 2017

Keywords:
Hybridizable discontinuous Galerkin method
Hybrid mesh
Time-harmonic Maxwell's equations
Electromagnetics

A B S T R A C T

This work presents a high-order hybridizable discontinuous Galerkin (HDG) method based on hybrid mesh to solve the two-dimensional time-harmonic Maxwell's equations. The hybrid mesh consists of unstructured triangular mesh in the domain with complex geometries and structured quadrilateral mesh in the rest domain. The coupling of different meshes is natural in HDG framework. Numerical simulations show that the HDG method on hybrid mesh converges at the optimal rate. One can save computation costs by employing hybrid mesh due to the reduction in number of degrees of freedom (DOFs).

1. Introduction

The discontinuous Galerkin (DG) [1] method had been employed to solve the time-harmonic Maxwell's equations in recent years because of the easy-obtained high order accuracy, natural parallelism and hp-adaptivity. However, it has one main drawback for stationary problems: to obtain a similar accuracy, the number of globally coupled DOFs is much larger than that of conforming finite element methods [2]. This leads to a large amount of memory consumption and computational time. Hybridization of DG method [3–5] alleviates this problem by introducing an additional hybrid variable living on the skeleton of the mesh only. Then numerical fluxes are defined in terms of this newly introduced hybrid variable. Finally, the HDG method produces a system of linear equations [6,7] only involving the DOFs of the additional hybrid variable [8]. The reduction in DOFs directly leads to less computational time and memory consumption [2,3].

In order to further reduce the number of DOFs, we employ the technique of hybrid unstructured/ structured meshes. Unstructured mesh is used to discretize the domains with complex geometries or material

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http://dx.doi.org/10.1016/j.aml.2016.12.018

compositions. While structured mesh is used for the simple domains [9,10]. Different meshes are glued together naturally under HDG discretization by sharing the hybrid variable on the interfaces between two different meshes. We show that using hybrid mesh one can easily deal with the geometric complexity while obtaining the same level of accuracy.

The rest of the paper is presented as follows. In Section 2, we introduce the two-dimensional time-harmonic Maxwell's equations, and provide some notations used in this paper. In Section 3, we propose the HDG formulations and state how to adapt it to hybrid mesh. Section 4 shows numerical results for the comparison between the HDG method on the hybrid mesh and the fully unstructured mesh. Finally, we draw some conclusions in Section 5.

2. Problem statement and notation

2.1. Time-harmonic Maxwell's equations

We consider the problem of electromagnetic wave propagating in a 2D domain Ω with a boundary $\partial\Omega$. It can be described by the time-harmonic Maxwell's equations:

$$\begin{cases} i\omega\varepsilon_r E - \mathrm{curl}\mathbf{H} = 0, & \text{in } \Omega, \\ i\omega\mu_r \mathbf{H} + \mathbf{curl}E = 0, & \text{in } \Omega, \\ E + (\mathbf{n}\times\mathbf{H}) = E^{\mathrm{inc}} + (\mathbf{n}\times\mathbf{H}^{\mathrm{inc}}) = g^{\mathrm{inc}}, & \text{on } \Gamma_a, \end{cases} \tag{1}$$

where i is the imaginary unit, ω is the angular frequency, ε_r and μ_r are the relative electric permittivity and magnetic permeability respectively, $\mathbf{n}$ is the outward unit norm vector on $\partial\Omega$, and the superscript $^{\mathrm{inc}}$ denotes the incident fields. Note that we consider the first order absorbing boundary conditions (ABC).

Eq. (1) and the subsequent formulations will be described under transverse electric (TE) mode. However, the formulations under transverse magnetic (TM) mode also can be obtained straightforwardly. The differential operators in this setting are $\mathbf{curl}E = (\partial_y E, \; -\partial_x E)$ and $\mathrm{curl}\mathbf{H} = \partial_x H_y - \partial_y H_x$, and the cross-product is $\mathbf{u}\times\mathbf{v} = u_x v_y - u_y v_x$.

2.2. Notations

The computational domain Ω is divided into two parts Ω_t and Ω_q containing triangular elements and quadrilateral elements respectively. We denote by K an element on Ω, by T_h the union of all the triangle elements in domain Ω_t, by Q_h the union of all the quadrangle elements in domain Ω_q, and let $K_h = T_h \bigcup Q_h$. For two neighboring elements K^+ and K^-, we denote by F the interface shared by the two elements, in particular, by F_{in} the interface shared by a triangular element and a quadrilateral element. Moreover, F_h denotes the union of all interior faces and boundary faces.

On an interior face, we define means (averages) $\{\cdot\}$ and jumps $[\![\cdot]\!]$ as follows [2]:

$$\begin{cases} \{\mathbf{v}\}_F = \dfrac{1}{2}(\mathbf{v}^+ \times \mathbf{v}^-), \\ \{v\}_F = \dfrac{1}{2}(v^+ \times v^-), \end{cases} \qquad \begin{cases} [\![\mathbf{n}\times\mathbf{v}]\!]_F = \mathbf{n}^+\times\mathbf{v}^+ + \mathbf{n}^-\times\mathbf{v}^-, \\ [\![v\mathbf{t}]\!]_F = v^+\mathbf{t}^+ + v^-\mathbf{t}^-, \end{cases}$$

where $\mathbf{n}^\pm$ denote the outward unit normal vectors on two neighboring elements $K^\pm$ respectively, $\mathbf{t}^\pm$ the unit tangent vectors on the boundaries $\partial K^\pm$ such that $\mathbf{t}^+\times\mathbf{n}^+ = 1$ and $\mathbf{t}^-\times\mathbf{n}^- = 1$, and $(\mathbf{v}^\pm, v^\pm)$ denote traces of $(\mathbf{v}, v)$ on F from the interior of $K^\pm$.

We introduce discontinuous finite element spaces:

$$V_h^p = \{v \in L^2(\Omega) | v|_K \in \mathbb{P}^p(K), \;\; \forall K \in K_h\}, \quad \mathbf{V}_h^p = \{\mathbf{v} \in L^2(\Omega) | \mathbf{v}|_K \in \{\mathbb{P}^p(K)\}^2, \;\; \forall K \in K_h\},$$