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On the Stability of a Model for the Cutting of Metal Plates by Means of Laser Beams

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Abstract

In a class of systems of balance laws in several space dimensions, we prove the stability of solutions with respect to variations in the flow and in the source. This class comprises a model describing the cutting of metal plates by means of laser beam.

Keywords: Nonlocal Balance Laws; Laser Cutting
2010 MSC: 35L65

1. Introduction

Following [3], we consider this system of n balance laws in several space dimensions:

$$\begin{cases} \partial_t u_i + \mathrm{div}_x\, \varphi_i(t,x,u_i,\vartheta * u) = \Phi_i(t,x,u_i,\vartheta * u) \\ u_i(0,x) = \bar u_i(x) \end{cases} \qquad i = 1,\ldots,n\,. \tag{1}$$

Here, $t \in [0,+\infty[$ is time, $x \in \mathbb{R}^N$ is the space coordinate and $u_1,\ldots,u_n$ are the unknowns. The function ϑ is a smooth function defined in $\mathbb{R}^N$ attaining values in $\mathbb{R}^{m\times n}$, so that

$$\vartheta \in \mathbf{C_c^2}(\mathbb{R}^N;\mathbb{R}^{m\times n})\,, \qquad \big(\vartheta * u(t)\big)(x) = \int_{\mathbb{R}^N} \vartheta(x-\xi)\, u(t,\xi)\, \mathrm{d}\xi\,, \qquad \big(\vartheta * u(t)\big)(x) \in \mathbb{R}^m\,.$$

Requirements on the flows φ_i, on the sources Φ_i and on the initial data $\bar u_i$ ensuring the well posedness of (1) are provided below.

A key property of system (1) is that the equations are coupled only through the nonlocal convolution term $\vartheta * u$. It is this feature that allows a well posedness and stability theory, although we are dealing with *systems* of balance laws in *several* space dimensions.

The driving example motivating (1) is a new model for the cutting of metal plates by means of a laser beam, presented in [3, Section 3], see also [2, 4]. However, (1) also comprises the model [7], see also [3, Section 4], devoted to the dynamics on a conveyor belt, as well as several crowd dynamics models, e.g. [1, 6, 8]. Theorem 2.3 below, applied to each of these cases, provides the stability of solutions with respect to perturbations of fluxes and sources.

2. Results

Throughout, $\mathrm{grad}_x\, f$ and $\mathrm{div}_x\, f$ denote the gradient and the divergence of f with respect to the space variable $x \in \mathbb{R}^N$. Throughout, we fix the non trivial time interval $\hat I = [0,\hat T]$. For any $k > 0$, we also denote $\mathcal{U}_k = [-k,\,k]$ and $\mathcal{U}_k^m = [-k,\,k]^m$.

Recall the definition of solution to (1), based on [9, Definition 1], and the well posedness result obtained in [3].

Definition 2.1 ([3, Definition 2.1]). *Let $\bar u \in \mathbf{L}^\infty(\mathbb{R}^N,\mathbb{R}^n)$. A map $u\colon \hat I \to \mathbf{L}^\infty(\mathbb{R}^N,\mathbb{R}^n)$ is a solution on $\hat I$ to (1) with initial datum $\bar u$ if, for $i = 1,\ldots,n$, setting for all $z \in \mathbb{R}$*

$$\tilde\varphi_i(t,x,z) = \varphi_i\big(t,x,z,(\vartheta * u)(t,x)\big) \qquad \textit{and} \qquad \tilde\Phi_i(t,x,z) = \Phi_i\big(t,x,z,(\vartheta * u)(t,x)\big)\,,$$

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