Applied Mathematics Letters 68 (2017) 157–162

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

Existence of non-radial solutions of an elliptic system

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ARTICLE INFO

Article history:
Received 20 September 2016
Received in revised form 9 December 2016
Accepted 9 December 2016
Available online 12 January 2017

Keywords:
Elliptic system
Variational method
Non-radial symmetry

ABSTRACT

In this paper, we study the following elliptic system

$$\begin{cases} -\Delta u = \mu_1 u^3 + \beta u v^2, & x \in \Omega \\ -\Delta v = \mu_2 v^3 + \beta u^2 v, & x \in \Omega \\ u, v > 0,\ x \in \Omega, \quad u = v = 0, \quad x \in \partial\Omega, \end{cases}$$

where $\Omega \subset \mathbb{R}^4$ is an annulus, $\mu_1, \mu_2 > 0$ and $\beta > 0$. By variational method, we prove that the system has a non-radial solution.

1. Introduction

In this paper, we consider the following elliptic system

$$\begin{cases} -\Delta u = \mu_1 u^3 + \beta u v^2, & x \in \Omega \\ -\Delta v = \mu_2 v^3 + \beta u^2 v, & x \in \Omega \\ u, v > 0,\ x \in \Omega, \quad u = v = 0, \quad x \in \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega = \{x \in \mathbb{R}^4 \mid \ r^2 < |x|^2 < (r+d)^2\}$ is an annulus of $\mathbb{R}^4$, $\mu_1,\ \mu_2 > 0$, $\beta > 0$, d is a fixed positive number.

System (1.1) arises in a binary mixture of Bose–Einstein condensates with two different hyperfine states, it is also used to study the solitary wave solutions of Gross–Pitaevskii equations. In recent decades, many researchers have been concerned with the previous system, they have proved the existence, uniqueness, multiplicity and limit properties of the solutions of the system, variational method and perturbation method are used, see [1–9] and references therein.

Many mathematicians have considered the following scalar equation

$$-\Delta u + u = u^p,\ x \in \Omega, \quad u > 0\ x \in \Omega, \quad u = 0 \ \ x \text{ on } \partial\Omega, \tag{1.2}$$

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http://dx.doi.org/10.1016/j.aml.2016.12.020
0893-9659/

where $\Omega \subset \mathbb{R}^N$ is a radially symmetric domain, and $1 < p < 2^* - 1$, where

$$2^* = \begin{cases} +\infty, & \text{if } N = 1, 2, \\ \dfrac{2N}{N-2}, & \text{if } N \geq 3. \end{cases}$$

When Ω is a ball or entire space, by the moving plane method, Gidas–Ni–Nirenberg [10,11] have proved that the positive C^2-solutions are radial symmetry. After that, several researchers are interested in the radially symmetric solutions of elliptic equations and systems. A natural question, if Ω is radially symmetric but not a ball, whether the scalar equations have non-radial solutions? Coffman [12], Li [13] and Lin [14] considered the non-radial solutions in annulus, they obtained the partially symmetric solutions of the equations. Non-radial type solutions can be called symmetry-breaking, furthermore there are several symmetry-breaking results of scalar equations and elliptic system, see [15] and references therein.

There is not many non-radial results of system (1.1), see [16,17] and references therein. In this paper, by delicate calculation, we get the non-radial solutions of system (1.1), that is

Theorem 1.1. *Let d be fixed and $r > d$ be large enough, then system (1.1) has a non-radial solution.*

This paper is organized as follows. In Section 2, we list some preliminaries and lemma, and in Section 3, we give the proof of the main result.

2. Some preliminaries and lemma

Firstly we define the working space $H = H_0^1(\Omega) \times H_0^1(\Omega)$ with the norm

$$\|(u, v)\| = \left(\int_\Omega |\nabla u|^2 + |\nabla v|^2 dx \right)^{1/2},$$

where $H_0^1(\Omega)$ is the classical Sobolev space.

The functional of system (1.1) is defined as

$$I(u, v) = \frac{1}{2} \int_\Omega (|\nabla u|^2 + |\nabla v|^2) dx - \frac{1}{4} \int_\Omega (\mu_1 u^4 + \mu_2 v^4) dx - \frac{1}{2} \beta \int_\Omega u^2 v^2 dx,$$

and we define the following quotation

$$J(u, v) = \frac{\left(\int_\Omega |\nabla u|^2 + |\nabla v|^2 dx \right)^2}{\int_\Omega \mu_1 u^4 + \mu_2 v^4 dx + 2\beta \int_\Omega u^2 v^2 dx}.$$

Now we define

$$\Lambda_0 = \{(u, v) \in H \setminus \{(0, 0)\} : u, v \text{ are radial functions}\},$$

and

$$\Lambda_2 = \{(u, v) \in H \setminus \{(0, 0)\} : u(x) = u(|y_1|, |y_2|), v(x) = v(|y_1|, |y_2|)\},$$

where $y_1 = (x_1, x_2)$ and $y_2 = (x_3, x_4)$. We define the minimum value

$$\lambda_0 = \inf_{(u,v) \in \Lambda_0} J(u, v),$$

and

$$\lambda_2 = \inf_{(u,v) \in \Lambda_2} J(u, v).$$

Next we list a lemma, see [13].