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Some novel solutions for the two-coupled nonlinear Schrödinger equations

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We study an integrable two-coupled nonlinear Schrödinger (NLS) system by using the method of linear transformation. With the aid of the traveling wave solutions of the classical NLS equation, we can construct forty-eight different types of solutions for the two-coupled NLS system. Finally, we use numerical methods to compute the stability eigenvalue of the steady-state solution.

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I. INTRODUCTION

The nonlinear Schrödinger equation (NLSE) plays a vital role in optical fiber communications [1–6]. The propagation of the optical solitons can usually be described by the NLS type equations [7–11]. The scalar optical soliton is governed by a single NLSE, while the vector optical soliton by coupled ones. There is few research on the vector optical soliton. The classical model for vector solitons in the nonlinear optical system is the coupled NLSE [1–3]

$$\begin{aligned} &\mathrm{i}q_{1,t} + \frac{1}{2}q_{1,xx} + \sigma(|q_1|^2 + \alpha|q_2|^2)q_1 = 0, \\ &\mathrm{i}q_{2,t} + \frac{1}{2}q_{2,xx} + \sigma(\alpha|q_1|^2 + |q_2|^2)q_2 = 0, \end{aligned} \tag{1}$$

where $\sigma = \pm 1$, $\alpha > 0$ and $\sigma = 1$ means focusing case, $\sigma = -1$ means defocusing case. However, the above equations are not completely integrable in general except $\alpha = 1$. When $\alpha = 1$, this system is the integrable Manakov equation [12]. When $\alpha = 2$, this system is not integrable. We find that if we add some additional terms $\bar{q}_1 q_2^2$ and $\bar{q}_2 q_1^2$ in Eq. (1) and choose $\sigma = -1$ to have

$$\begin{aligned} &\mathrm{i}\frac{\partial}{\partial t}q_1 + \frac{1}{2}\frac{\partial^2}{\partial x^2}q_1 - |q_1|^2 q_1 - 2|q_2|^2 q_1 + \bar{q}_1 q_2^2 = 0, \\ &\mathrm{i}\frac{\partial}{\partial t}q_2 + \frac{1}{2}\frac{\partial^2}{\partial x^2}q_2 - 2|q_1|^2 q_2 - |q_2|^2 q_2 + \bar{q}_2 q_1^2 = 0, \end{aligned} \tag{2}$$

then Eq. (2) is integrable. The new additional term denotes the coherent coupling in the nonlinear optical model[14–21]. Meanwhile, Eq. (2) can be derived from the BEC [22, 23].

Since the significance in the physics, there are lots of research work on Eq. (2). For instance, the Painlevé analysis, Lax pair and soliton solution were given in [13], the Darboux transformation and conservation laws were constructed in [18], and the two-bright solitons were analyzed in [21]. However, most of previous studies concentrate on the focusing case and the method to solve this model is rather complicated. In this work, we will focus on the defocusing case and provide a simple method to obtain some exact solutions.

The paper is organized as the following: In section II, we will exhibit the explicit method which could decompose the coupled Eq. (2) into two independent classical NLSEs. Since the exact solutions for the classical NLSE are well known, then we can construct some interesting exact solutions which have never been reported before. In section III, based on the aforementioned method and two different types of solution for the classical NLSE, we can construct forty-eight different kinds of solution for the coupled NLSE (2), and plot some figures to exhibit the explicit dynamics for these exact solutions, we show some figures by computer plotting. In section IV, the stability for some traveling wave solutions is given by the linear stability theory. In the final section, we will give some discussions and conclusions.

II. DECOMPOSING CNLSE INTO TWO INDEPENDENT NLSES

Generally speaking, it is not easy to find the solution of nonlinear partial differential equations directly. There are lots of methods to look for exact solutions, for instance, inverse scattering method [7], Darboux transformation[24, 25]

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