Periodic solutions of Liebau-type differential equations

Fang-Fang Liao

PII: S0893-9659(17)30038-1
DOI: http://dx.doi.org/10.1016/j.aml.2017.02.001
Reference: AML 5178

To appear in: *Applied Mathematics Letters*

Received date : 26 December 2016
Revised date : 2 February 2017
Accepted date : 3 February 2017

PERIODIC SOLUTIONS OF LIEBAU-TYPE DIFFERENTIAL EQUATIONS

FANG-FANG LIAO

ABSTRACT. We study the existence of positive periodic solutions for a second order differential equation which is related to the Liebau phenomenon. The proof is based on the fixed point theorem in cones. Our results improve and generalize those in the literature.

1. Introduction

In [13], Propst presented differential equations modeling a periodically forced flow through different pipe-tank configurations. Such differential equations are related to the Liebau phenomenon, which was observed by the physician Liebau [12] in the 1950s when he tried to developed experiments dealing with a valveless pumping phenomenon arising in blood circulation, and was numerically investigated in [1]. In particular, from the mathematical point of view, Propst presented the following interesting differential equation

$$u'' + au' = \frac{1}{u}(e(t) - bu') - c, \tag{1.1}$$

where $a \geq 0, b > 1, c > 0$ are constants and $e \in \mathbb{C}(\mathbb{R}/T\mathbb{Z}, \mathbb{R})$. For the reason of periodic force, it is natural to look for the T-periodic solutions of (1.1) under suitable conditions. Although equation (1.1) is singular, fortunately it can be transformed into a regular equation by the change of variables, which has been used in [5, 6, 7]. In fact, equation (1.1) is equivalent to the following equation

$$x'' + ax' = \frac{e(t)}{\mu}x^{1-2\mu} - \frac{c}{\mu}x^{1-\mu}, \tag{1.2}$$

by taking the change of variables $u = x^{\mu}$ with $\mu = \frac{1}{b+1}$.

In the recent papers [5, 6], the following generalized equation of the form (1.2) was studied

$$x'' + ax' = r(t)x^{\alpha} - s(t)x^{\beta}, \tag{1.3}$$

in which $a \geq 0$, $r, s \in \mathbb{C}(\mathbb{R}/T\mathbb{Z}, \mathbb{R})$ and $0 < \alpha < \beta < 1$. By using a well-known fixed point theorem together with a careful analysis of the Green's functions of the linear equations, the existence of positive T-periodic solutions of (1.3) has been established. In particular, as an application, explicit existence conditions were given

2000 *Mathematics Subject Classification.* Primary 34B15.

Key words and phrases. Periodic solutions; Liebau-type differential equations; Fixed point theorem in cones.

Fang-Fang Liao was supported by QingLan project of Jiangsu Province and the National Natural Science Foundation of China (Grant No. 11461016).