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Identification of non-smooth boundary heat dissipation by partial boundary data

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ABSTRACT

When electronic devices are in operation, the sharp change of the temperature on devices surface can be considered as an indicator of devices faults. Based on this engineering background, we consider an inverse problem for 1-dimensional heat conduction model, with the aim of detecting the nonsmooth heat dissipation coefficient from measurable temperature on the device surface. We establish the uniqueness for the nonsmooth dissipation coefficient and prove the convergence property of the minimizer of the regularizing cost functional for the inverse problem theoretically. Then a double-iteration scheme minimizing the data-match term and the regularization term alternatively is proposed to implement the reconstructions.
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1. Introduction

Consider the working process for supercomputers. As the CPU operates for higher performance computing, the core of the CPU becomes quite hot. The heat transfer process for this configuration can be modeled by the following 1-dimensional parabolic system [1]

$$\begin{cases} K_s\dfrac{\partial^2 T(x,t)}{\partial x^2} = \rho C_p\dfrac{\partial T(x,t)}{\partial t}, & 0<x<L, t>0, \\ -K_s\dfrac{\partial T(0,t)}{\partial x} = q(t), & t>0, \\ K_s\dfrac{\partial T(L,t)}{\partial x} = h(t)(T_\infty - T(L,t)), & t>0, \\ T(x,0) = T_0, & 0\le x\le L, \end{cases} \tag{1.1}$$

where $(K_s, \rho, C_p, T_\infty)$ are the related physical constants, $q(t)$ is the heat flux acting on the exterior of the thermal conductor, and $h(t)$ is the heat dissipation coefficient on the surface $x = L$, representing the dynamic exchanges of thermal energy with the environment around.

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When there is something wrong inside CPU core, the temperature on the device surface $x = L$ will have some sharp jumps, leading to the nonsmooth change of the heat dissipation coefficient. Therefore the dynamical monitoring of $h(t)$ from other observable heat information is a potential way to the on-line identification of the drawback of CPU core. By such engineering motivations, we consider the following heat conduction problem after re-scaling the physical parameters $(K_s, \rho, C_p, T_\infty)$:

$$\begin{cases} u_t - a^2 u_{xx} = 0, & (x,t) \in (0,\pi) \times (0,T) := Q_T, \\ u_x(0,t) = f(t), \quad u_x(\pi,t) + \sigma(t)u(\pi,t) = \varphi(t), & t \in (0,T], \\ u(x,0) = u_0(x), & x \in [0,\pi] \end{cases} \tag{1.2}$$

for known $(f(t), \varphi(t), u_0(x))$. If we take $u(x,t) = T(x,t) - T_\infty$ for $a^2 = \frac{\rho C_p}{K_s}$ and $\varphi = 0, \sigma(t) = \frac{h(t)}{K_s}$, then (1.2) becomes (1.1), i.e., the system (1.1) can be considered as a special case of our general problem (1.2). We want to identify the nonsmooth heat transfer coefficient $\sigma(t)$ from the boundary observation $u(0,t) := H(t)$ or its noisy approximation $H^\delta(t)$ satisfying

$$\|H^\delta(\cdot) - H(\cdot)\|_{L^2(0,T)} \le \delta, \quad t \in (0,T] \tag{1.3}$$

with noisy level $\delta > 0$. Mathematically, this problem belongs to the category of boundary reconstructions for parabolic system for which the related works can be found in [2–5] and the references therein. However, for nonsmooth $\sigma(t)$, such kinds of problems are rarely studied, with the main difficulty that some balances between catching the jump of σ and stabilizing the approximate solution should be kept.

After proving the uniqueness of this inverse problem for nonsmooth $\sigma(t)$ in Section 2, we propose to recover σ by Tikhonov regularizing scheme with total variation penalty term. Instead of optimizing the regularizing functional containing the data matching term and the regularizing term, we solve this problem in Section 3 by optimizing these two terms alternatively with inner–outer recursions, where the outer recursion optimizes the data matching term only once at each step, while the inner recursion optimizes the regularizing term several times to catch the jump points of nonsmooth solution. Such a double-recursion strategy avoids both the choice strategy of the regularizing parameter and the non-differentiability of total variation penalty term. The numerical implementations for our scheme are given.

2. Uniqueness and reconstruction by optimization

Due to our engineering motivations, we should establish the unique reconstruction of $\sigma(t)$ in $L^2(0,T)$, which is completely different from the uniqueness in [5], where the maximum principle is applicable. Compared with the uniqueness result in [5], the proof should be much changed, since we consider the boundary impedance in $L^2(0,T)$, not in $C[0,T]$. Introduce the admissible set

$$\Sigma' := \{\sigma(t) : \sigma \in L^2(0,T), \sigma(0) = \sigma_*, 0 < \sigma_- \le \sigma(t) \le \sigma_+ \text{ a.e. in } (0,T)\}.$$

Theorem 2.1. *Assume that $0 < -f(t), \varphi(t) \in C[0,T]$ and $0 < u_0 \in C[0,\pi]$. Then for any $\sigma \in \Sigma'$, there exists a unique solution $u[\sigma](x,t)$ to (1.2) in $C(\overline{\Omega}_T)$. Moreover, if $u[\sigma_1](0,t) = u[\sigma_2](0,t)$ in $C[0,T]$ for $\sigma_1, \sigma_2 \in \Sigma'$, then $\sigma_1 = \sigma_2$ in $L^2(0,T)$.*

Proof. By modifying the proof for Theorem 1.2 in Chapter 1 in [6] slightly, we know that the unique solution $u[\sigma](x,t)$ to (1.2) is in $C(\overline{\Omega}_T)$ for $\sigma \in \Sigma'$ and known smooth data $(u_0(x), f(t), \varphi(t))$ satisfying the compatible conditions, noticing that the potential representation of the solution constructed here is also continuous for $\sigma \in L^2(0,T)$.

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