Applied Mathematics Letters 69 (2017) 82–86

Contents lists available at ScienceDirect

Applied Mathematics Letters

www.elsevier.com/locate/aml

The lower bound for the blowup time of the solution to a quasi-linear parabolic problem

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ARTICLE INFO

Article history:
Received 14 November 2016
Received in revised form 5 January 2017
Accepted 5 January 2017
Available online 9 February 2017

Keywords:
Quasi-linear parabolic equation
Boundary condition
Lower bound for blowup time

ABSTRACT

In this paper, using a delicate application of general Sobolev inequality, we establish the lower bound for the blowup time of the solution to a quasi-linear parabolic problem, which improves the result of Theorem 2.1 in Bao and Song (2014).

1. Introduction and motivation

In this paper, we will establish the lower bound for the blowup time of the solution to the following problem:

$$\begin{cases} u_t = \Delta G(u) + f(u) & \text{in } \Omega \times (0,T) \\ u(x,t) = 0 \quad \text{or} \quad \dfrac{\partial u(x,t)}{\partial \nu} = 0 & \text{on } \partial\Omega \times (0,T) \\ u(x,0) = g(x) \geq 0 & \text{in } \Omega. \end{cases} \tag{1.1}$$

Here $\Omega \subset \mathbb{R}^n (n \geq 3)$ is a smooth bounded domain, ν is the outward norm vector. $g(x)$ is a continuous nonnegative function and satisfies the compatible condition. The functions $f(s)$ and $G(s)$ satisfy the following assumptions:

Assumption (f). (f_1): $f(s) \geq 0$ for $s \geq 0$;
(f_2): $\int_s^\infty \frac{d\eta}{f(\eta)}$ is bounded for all $s \geq s_0 > 0$;

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http://dx.doi.org/10.1016/j.aml.2017.01.004
0893-9659/

(f_3): There exist positive constants $0 < \gamma < 1$, $k > \max\{\frac{4(n-2)-n\gamma}{2n}, \frac{2}{n}\}$ and β such that

$$f(s)\Big(\int_s^\infty \frac{\mathrm{d}\eta}{f(\eta)}\Big)^{nk+1} \to \infty \quad \text{as } s \to 0^+$$

and

$$f'(s)\int_s^\infty \frac{\mathrm{d}\eta}{f(\eta)} \le nk+1-\beta \quad \text{for } s \ge 0.$$

Assumption (G). (G_1): $G(s) \ge 0$ and $G'(s) \ge 0$ for $s \ge 0$.

(G_2): There exists a positive constant K such that $G'(s) \ge K(\int_s^\infty \frac{\mathrm{d}\eta}{f(\eta)})^{-\gamma}$ for $s > 0$.

(1.1) can be used to describe the ignite model, while u represents the temperature of the combustion. If $G(u) = u^m$ and $f(u) = u^p$ in (1.1), the equation $u_t = \Delta u^m + u^p$ is the porous medium equation. By the results of [1–3], the solution to (1.1) will blow up in finite time and the blowup set has positive measure under the Assumptions (f), (G) and other conditions. However, we do not discuss the blowup condition and the blowup set, we just deal with the lower bound for the blowup time of the blowup solution to (1.1) in this paper. About the topic on the lower bound for the blowup time of the solution to (1.1), we can also refer [4–9] and the references therein to know more information.

The direct motivation of this paper is [10], where Bao and the second author of this paper only established the lower bound for the blowup time of the solution to (1.1) subject to Dirichlet boundary condition. Naturally, we hope to get the lower bound for the blowup time of the solution to (1.1) subject to Neumann boundary condition. Inspired by Payne–Schaefer's idea, we will use a delicate application of general Sobolev inequality to deal with both (1.1) subject to Neumann boundary condition and (1.1) subject to Dirichlet boundary condition.

We will give the details to obtain the lower bound for the blowup time of the solution to (1.1) in the next section.

2. Lower bounds for the blowup time

In this section, inspired by Payne–Schaefer's idea and using a delicate application of general Sobolev inequality, we will establish the lower bound for the blowup time of the solution to (1.1).

Let

$$\phi(t) = \int_\Omega v^{nk}(u(x,t))\mathrm{d}x, \quad v(u(x,t)) = \left(\int_{u(x,t)}^\infty \frac{\mathrm{d}\eta}{f(\eta)}\right)^{-1} \tag{2.1}$$

and assume that

$$\lim_{t\to T^-} \phi(t) = +\infty. \tag{2.2}$$

Then

$$\begin{aligned}
\phi'(t) &= \int_\Omega \frac{-nk}{\left(\int_u^\infty \frac{\mathrm{d}\eta}{f(\eta)}\right)^{nk+1}} \cdot \left(-\frac{1}{f(u)}\right) u_t \mathrm{d}x \\
&= \int_\Omega \frac{nk}{\left(\int_u^\infty \frac{\mathrm{d}\eta}{f(\eta)}\right)^{nk+1}} \cdot \left(\frac{1}{f(u)}\right) [\Delta G(u) + f(u)]\mathrm{d}x \\
&= nk \int_\Omega \left[-\frac{(nk+1)v^{nk+2}}{[f(u)]^2} G'(u)|\nabla u|^2 + \frac{v^{nk+1}}{[f(u)]^2} f'(u)G'(u)|\nabla u|^2\right] \mathrm{d}x
\end{aligned}$$