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Bérangère Delourme, Patrick Joly, Elizaveta Vasilevskaya

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Existence of guided waves due to a lineic perturbation of a 3D periodic medium

Bérangère Delourme[a,b], Patrick Joly[b], Elizaveta Vasilevskaya[a,b]

a: Université Paris 13, Sorbonne Paris Cité, LAGA, UMR 7539, 93430 Villetaneuse, France
b: POEMS, UMR 7231 CNRS/ENSTA/Inria, ENSTA ParisTech, 828 boulevard des Maréchaux, 91762 Palaiseau Cedex, France

Abstract

In this note, we exhibit a three dimensional structure that permits to guide waves. This structure is obtained by a geometrical perturbation of a 3D periodic domain that consists of a three dimensional grating of equi-spaced thin pipes oriented along three orthogonal directions. Homogeneous Neumann boundary conditions are imposed on the boundary of the domain. The diameter of the section of the pipes, of order $\varepsilon > 0$, is supposed to be small. We prove that, for ε small enough, shrinking the section of one line of the grating by a factor of $\sqrt{\mu}$ $(0 < \mu < 1)$ creates guided modes that propagate along the perturbed line. Our result relies on the asymptotic analysis (with respect to ε) of the spectrum of the Laplace-Neumann operator in this structure. Indeed, as ε tends to 0, the domain tends to a periodic graph, and the spectrum of the associated limit operator can be computed explicitly.

Keywords : guided waves, periodic media, spectral theory.

AMS codes : 78M35, 35J05, 58C40

1 Statement of the problem and objective

1.1 Description of the problem

Let ω_1, ω_2 and ω_3 be three Lipschitz bounded domains of $\mathbb{R}^2$ of same area ($|\omega_1| = |\omega_2| = |\omega_3|$) containing the origin $(0,0)$, let $\varepsilon > 0$ be a parameter (that is going to be small), and let a_1, a_2 and a_3 be three positive real numbers. We denote by $(\mathbf{e}_i)_{i\in\{1,2,3\}}$, the standard basis of $\mathbb{R}^3$. For any $(k,\ell) \in \mathbb{Z}^2$, we consider the three dimensional domain $D^\varepsilon_{k,\ell,3}$ defined by

$$D^\varepsilon_{k,\ell,3} = \left\{(x_1,x_2,x_3) \in \mathbb{R}^3 \text{ such that } \big((x_1 - a_1 k)/\varepsilon, (x_2 - a_2\ell)/\varepsilon\big) \in \omega_3\right\},$$

which is an unbounded cylinder of constant cross section $\varepsilon\omega_3$. It is infinite along the $\mathbf{e}_3$ direction (invariant with respect to x_3) and contains the point $(a_1 k, a_2 \ell, 0)$. Similarly, for any $(k,\ell) \in \mathbb{Z}^2$, we define the domains

$$D^\varepsilon_{k,\ell,1} = \left\{(x_1,x_2,x_3) \in \mathbb{R}^3 \text{ such that } \big((x_2 - a_2 k)/\varepsilon, (x_3 - a_3\ell)/\varepsilon\big) \in \omega_1\right\},$$
$$D^\varepsilon_{k,\ell,2} = \left\{(x_1,x_2,x_3) \in \mathbb{R}^3 \text{ such that } \big((x_1 - a_1 k)/\varepsilon, (x_3 - a_3\ell)/\varepsilon\big) \in \omega_2\right\},$$

and we consider the periodic domain Ω_ε given by

$$\Omega_\varepsilon = \bigcup_{i\in\{1,2,3\}} \bigcup_{(k,\ell)\in\mathbb{Z}^2} D^\varepsilon_{k,\ell,i}. \tag{1}$$

The domain Ω_ε is a three dimensional grating of equi-spaced parallel pipes (of constant cross section) oriented along the three orthogonal directions $\mathbf{e}_1$, $\mathbf{e}_2$ and $\mathbf{e}_3$. It is a_j-periodic with respect to x_j, $j = 1,2,3$. Moreover, the points $(ka_1, \ell a_2, ma_3)$, $(k,\ell,m) \in \mathbb{Z}^3$, belong to Ω_ε.

In order to create guided modes, we introduce a linear defect (see [1]-[5]-[2]) in the periodic structure by modifying the section size of one pipe of the grating (it is conjectured that guided modes cannot appear in the purely periodic structure, see [4] for the proof in the case of a symmetric medium). More precisely, we assume that the domain $D^\varepsilon_{0,0,3}$ is replaced with the domain

$$D^{\varepsilon,\mu}_{0,0,3} = \left\{(x_1,x_2,x_3) \in \mathbb{R}^3 \text{ such that } \big(x_1/(\sqrt{\mu}\varepsilon), x_2/(\sqrt{\mu}\varepsilon)\big) \in \omega_3\right\},$$