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Necessary and sufficient conditions for the nonexistence of limit cycles of Leslie–Gower predator–prey models

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ABSTRACT

In this paper we consider a predator–prey model with Leslie–Gower functional response. We present the necessary and sufficient conditions for the nonexistence of limit cycles by the application of the generalized Dulac theorem. As a result, we give the necessary and sufficient conditions for which the local asymptotic stability of the positive equilibrium implies the global stability for this model. Our results extend and improve the results presented by Aghajani and Moradifam (2006) and Hsu and Huang (1995).

1. Introduction

In papers [1,2], Leslie introduced a predator–prey model where the "carrying capacity" of the predator's environment is proportional to the number of prey. Leslie stresses the fact that there are upper limits to the rates of increase of both prey and predator, which are not recognized in the Lotka–Volterra model. These upper limits can be approached under favourable conditions: for the predator, when the number of prey per predator is large; for the prey, when the number of predators (and perhaps the number of prey also) is small.

In the case of continuous time, these considerations lead to the following differentiable equation model:

$$\begin{cases} \dot{x} = rx\left(1 - \dfrac{x}{k}\right) - yp(x), \\ \dot{y} = y\left(\delta - \beta\dfrac{y}{x}\right), \\ x(0) > 0,\ y(0) > 0. \end{cases} \tag{1}$$

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In system (1), variables $x(t)$ and $y(t)$ stand for prey and predator population (or densities), respectively, as functions of time. The parameters r, δ are the intrinsic growth rates of prey and predator, respectively. k is the carrying capacity of the prey; $p(x)$ is a predator response function; β/δ is a measure of the food equality that the prey provides for conversion into predator births. r, k, δ and β are positive constants. Moreover, the function $p(x)$ satisfies the following assumptions:

(A_1) $p(0) = 0$,

(A_2) $p'(x) > 0$, for $x > 0$.

The form of the predator equation in system (1) was first introduced by Leslie [1,2]. The term $\delta x/\beta$ of this equation is called the Leslie–Gower term. This interesting formulation for the predator dynamics has been discussed by Leslie and Gower [3] and Pielou [4].

Notice that the point $(k, 0)$ is an equilibrium of system (1). The hypotheses (A_1) and (A_2) immediately imply that the graph of $y = \frac{rx(1-\frac{x}{k})}{p(x)}$, and $y = \frac{\delta x}{\beta}$, has a unique intersection x_* satisfying $0 < x_* < k$. Thus, system (1) possesses a unique positive equilibrium $E_* = (x_*, y_*)$, where

$$y_* = \frac{rx_*(1-\frac{x_*}{k})}{p(x_*)} = \frac{\delta}{\beta}x_*.$$

Let $F(x) = \frac{rx(1-\frac{x}{k})}{p(x)}$, $\varphi(x) = \frac{\beta F(x)}{x}$, $\psi(x) = \frac{p(x)}{x}$,

$G(x) = \frac{\frac{d}{dx}(p(x)F'(x))}{\varphi'(x)}$, $H(x) = \frac{\frac{d}{dx}(p^2(x)F'(x)) + \frac{\beta r}{k}}{\delta p'(x) + \frac{\beta r}{k}}$.

The existence, uniqueness and nonexistence of limit cycles are the most important problems related to two-dimensional predator–prey systems (see [5,6]). Many authors have studied the nonexistence problem for limit cycles and served interesting criteria are given in [7,8]. Recently Aghajani and Moradifam [7] presented the following result for the nonexistence of limit cycles of system (1).

Theorem 1.1. *Suppose that the assumptions (A_1) and (A_2) hold. Moreover, $p'(0) \geq \frac{r}{y_*}$. If one of the following conditions holds:*

(i) *$\psi''(x)$ has no zero in $(0, k)$,*

(ii) *$F'(0) > 0$ and $\psi''(x)$ has at most one zero in $(0, k)$,*

then system (1) has no limit cycles.

However, Theorem 1.1 and the previous results [8] are inapplicable to the following system

$$\begin{cases} \dot{x} = rx\left(1 - \dfrac{x}{k}\right) - yp(x), \\ \dot{y} = y\left(\delta - \beta\dfrac{y}{x}\right), \\ x(0) > 0,\ y(0) > 0, \end{cases} \tag{2}$$

where $p(0) = 0$, and $p(x) = e^{-\frac{a}{x}}$ for $x > 0$ and $a > 0$. We call $p(x)$ the inverted Ivlev's response function because it is obtained by inverting the growth rates of Ivlev's function $1 - e^{-\frac{a}{x}}$ at 0 and $+\infty$.

In this paper we extend and improve Theorem 1.1 and the previous results presented in [8]. We derive some new necessary and sufficient conditions for the nonexistence of limit cycles of system (1) by using generalized Dulac theorem. Moreover, by Poincaré–Bendixson theorem, we give the necessary and sufficient conditions for which the local asymptotic stability of the positive equilibrium implies the global stability for this model. Our results can be applied to system (2) for the nonexistence of limit cycles.

2. Main results

In this section we present the necessary and sufficient conditions for the nonexistence of limit cycles of system (1) by using generalized Dulac theorem. We also give the necessary and sufficient conditions for