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One-Dimensional Model of a Suspension Bridge: Revision of Uniqueness Results

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Abstract

This paper brings a revision of the so far known uniqueness result for a one-dimensional damped model of a suspension bridge. Using standard techniques, however with finer arguments, we provide a significant improvement and extension of the allowed interval for the stiffness parameter.

Keywords: suspension bridge, beam equation, unique weak solution

2000 MSC: 35B10, 35D05, 70K30

1. Introduction

We consider a nonlinear one-dimensional model of a suspension bridge introduced by Lazer and McKenna [7] and studied later in many papers (e.g., [1, 2, 3, 4, 5, 6, 8]):

$$\begin{aligned} &mu_{tt} + EIu_{xxxx} + bu_t + \kappa u^+ = h(x,t),\\ &u(0,t) = u(L,t) = u_{xx}(0,t) = u_{xx}(L,t) = 0,\\ &u(x,t+2\pi) = u(x,t),\ -\infty < t < +\infty,\ x \in (0,L), \end{aligned} \tag{1}$$

or its rescaled form, respectively,

$$\begin{aligned} &u_{tt} + \alpha^2 u_{xxxx} + \beta u_t + ku^+ = h(x,t),\\ &u(0,t) = u(\pi,t) = u_{xx}(0,t) = u_{xx}(\pi,t) = 0,\\ &u(x,t+2\pi) = u(x,t),\ -\infty < t < +\infty,\ x \in (0,\pi). \end{aligned} \tag{2}$$

This model represents the bridge as a damped beam with simply supported ends, subject to a periodic external force and to the nonlinear restoring force of cables hanging on a solid frame. The displacement $u(x,t)$ is measured as positive in the downward direction and the cables are taken as one-sided springs obeying Hooke's law, with a restoring force proportional to the displacement if they are stretched, and with no restoring force if they are compressed. We recall that $u^+(x,t) = \max\{0, u(x,t)\}$ is the positive part of $u(x,t)$ and k (or κ, respectively) can be interpreted as the stiffness of the cables. The meaning of other parameters can be found, e.g., in [2]. Evidently, only $\alpha > 0$, $\beta > 0$ and $k > 0$ make sense from the physical point of view, however, for the sake of generality, we will deal with $k \in \mathbb{R}$ throughout the text.

The aim of this paper is to revise the original result of [9], which says that for sufficiently small $|k|$, the problem (2) admits a unique solution for any right-hand side. Using the same techniques, however with finer arguments, we provide a significant improvement and extension of the allowed values of k. This means that even for a more pronounced asymmetry, the system possesses a unique solution for any loading and no bifurcations can occur.

2. Abstract setting

Let us denote by $\Omega = (0,\pi) \times (0, 2\pi)$ the considered domain and by $H = L^2(\Omega, \mathbb{R})$ the real Hilbert space equipped with the standard scalar product and the corresponding norm. Further, we denote by $\mathcal{D}$

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