Accepted Manuscript

Existence and uniqueness of positive periodic solutions for a class of integral equations with mixed monotone nonlinear terms

Shugui Kang

PII: S0893-9659(17)30093-9
DOI: http://dx.doi.org/10.1016/j.aml.2017.03.013
Reference: AML 5211

To appear in: *Applied Mathematics Letters*

Received date : 29 January 2017
Revised date : 19 March 2017
Accepted date : 19 March 2017

Existence and uniqueness of positive periodic solutions for a class of integral equations with mixed monotone nonlinear terms *

Shugui Kang
College of Mathematics and Computer Science, Shanxi Datong University, Datong, Shanxi 037009, P. R. China

Abstract

We study the existence and the uniqueness of positive periodic solutions for a class of integral equation of the form

$$\phi(x) = \int_{[x,x+\omega]\cap G} K(x,y)[f_1(y,\phi(y-\tau(y))) + f_2(y,\phi(y-\tau(y)))]dy,\ \ x \in G,$$

where G is a closed subset of $\mathbb{R}^N$ with periodic structure. Our analysis relies on the fixed point theory for mixed monotone operator in Banach space.

key words: periodic solution, fixed point theory, mixed monotone operator, existence and uniqueness

Mathematics Subject Classifications: 39A23

1 Introduction

We first introduce some notations that will be used in this paper. Let $\mathbb{R}^N$ be the N-dimensional Euclidean space endowed with the componentwise ordering '$\le$'. For any $u, v \in \mathbb{R}^N$, the 'interval' $[u, v]$ denotes the set $\left\{x \in \mathbb{R}^N \mid u \le x \le v\right\}$. Let $\omega = (\omega_1, ..., \omega_N) \in \mathbb{R}^N$ with positive components and $e^{(1)} = (1, 0, ...0), ..., e^{(N)} = (0, ..., 0, 1)$ be the standard orthonormal vectors of $\mathbb{R}^N$. It is assumed that G is a closed subset of $\mathbb{R}^N$ which has the following 'periodic' structure: for each $x \in G$,

$$x + \omega_i e^{(i)} \in G, \qquad i = 1, 2, ..., N,$$

and for each pair $y, z \in G$,

$$\mu\left([y, y + \omega] \cap G\right) = \mu\left([z, z + \omega] \cap G\right) > 0.$$

For the sake of convenience, we set

$$G(x) = [x, x + \omega] \cap G.$$

It is well-known that the nonlinear Hammerstein integral equation of the form

$$\phi(x) = \int_\Omega B(x, y) f(y, \phi(y))dy$$

*Email: dtkangshugui@126.com