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Growth Rates of Solutions of Superlinear Ordinary Differential Equations

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Abstract

In this letter, we obtain sharp estimates on the growth rate of solutions to a nonlinear ODE with a nonautonomous forcing term. The equation is superlinear in the state variable and hence solutions exhibit rapid growth and finite–time blow–up. The importance of ODEs of the type considered here stems from the key role they play in understanding the asymptotic behaviour of more complex systems involving delay and randomness.

Keywords: Nonlinear, ordinary differential equations, superlinear, growth rates, unbounded solutions
2010 MSC: 34C11, 34E10

1. Introduction

We study the asymptotic behaviour of rapidly growing solutions to the nonlinear ordinary differential equation

$$x'(t) = f(x(t)) + h(t), \quad t \geq 0; \quad x(0) = \psi > 0. \tag{1.1}$$

Rapid growth, and possibly even finite–time blow–up, of solutions is ensured by assuming

$$f \in C((0,\infty);(0,\infty)), \quad f \text{ is increasing}, \quad x \mapsto f_1(x) := f(x)/x \text{ is ultimately increasing}, \quad \lim_{x\to\infty} f_1(x) = \infty. \tag{f}$$

Note that (f) *precludes* f *being subadditive* (cf. [1]). Assuming f is locally Lipschitz continuous is sufficient to ensure a unique solution to (1.1) and, in order to simplify matters, we do so henceforth. We also assume

$$h \in C((0,\infty);\mathbb{R}), \quad H(t) = \int_0^t h(s)\,ds \geq 0 \text{ for each } t \geq 0. \tag{H}$$

While understanding the asymptotics of (1.1) is undoubtedly interesting in its own right, our primary interest in (1.1) stems from the key role it plays in more complex systems exhibiting rapid growth. The asymptotic behaviour of blow–up solutions of nonlinear Volterra equations, such as

$$x'(t) = \int_0^t w(t-s)f(x(s))\,ds + h(t), \quad t \geq 0; \quad x(0) = \psi > 0, \tag{1.2}$$

have attracted considerable attention (see [2, 3, 4] and the references therein). Of particular interest is the behaviour of solutions to (1.2) in the key limit, if explosion occurs, or for large times, if solutions are global; the results of this letter for the simpler equation (1.1) are an important first step in such an analysis (see e.g. [5] for sublinear equations). Similarly, the nonlinear stochastic differential equation

$$X(t) = \psi + \int_0^t f(X(s))\,ds + \int_0^t \sigma(s)\,dB(s), \quad t \geq 0, \tag{1.3}$$

can be studied using the results of this note (see Corollary 3 and [6] for analysis in the sublinear case). Finally, we remark in the case that a is a positive and continuous function, the non–autonomous ODE $z'(t) = a(t)f(z(t)) + h(t)$ can be analysed by similar methods, since $\tilde{x}(t) = z(A^{-1}(t))$ obeys (1.1), where $A(t) = \int_0^t a(s)\,ds \to \infty$ as $t \to \infty$, and $\tilde{H}(t) = H(A^{-1}(t))$. Similar time–rescaling can be applied to non–autonomous analogues of (1.3).

Equation (1.1) can be thought of as a perturbed version of the autonomous ODE

$$y'(t) = f(y(t)), \quad t \geq 0; \quad y(0) = \psi > 0, \tag{1.4}$$

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